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Global exponential stability for switched memristive neural networks with time-varying delays

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ABSTRACT

This paper considers the problem of exponential stability for switched memristive neural networks (MNNs) with time-varying delays. Different from most of the existing papers, we model a memristor as a continuous system, and view switched MNNs as switched neural networks with uncertain time-varying parameters. Based on average dwell time technique, mode-dependent average dwell time technique and multiple Lyapunov–Krasovskii functional approach, two conditions are derived to design the switching signal and guarantee the exponential stability of the considered neural networks, which are delay-dependent and formulated by linear matrix inequalities (LMIs). Finally, the effectiveness of the theoretical results is demonstrated by two numerical examples.

1. Introduction

Since the experimental prototyping of the memristor (Chua, 1971) was announced by the HP Lab (Strukov, Snider, Stewart, & Williams, 2008), memristive neural networks (MNNs) have been extensively investigated for their potential applications in different fields such as combinatorial optimization, brain emulation, knowledge acquisition and pattern recognition (Chen, Li, Huang, Chen, & Wang, 2014; Itoh & Chua, 2009; Pershin & Ventra, 2010). In particular, stability properties of such neural networks play a significant role in their designs for solving practical problems. For example, when a neural network is aimed to solve some type of optimization problem, one should guarantee that this neural network is globally asymptotically stable. However, in the implementation of neural networks, time delay is unavoidable due to finite switching speed of the amplifiers and communication time. The existence of time delays often causes instability, divergence or oscillation in neural networks. A variety of techniques have been developed to reduce the conservatism of stability conditions, such as descriptor model transformation approach (Fridman, 2001), improved bounding technique (Moon, Park, Kwon, & Lee, 2001), free-weighting matrix theory (Wu, He, She, & Liu, 2004), integral inequality technique (Huang, Cao, & Huang, 2005; Kwon, Lee, Park, & Cha, 2012). Using such methods, considerable efforts have been devoted to stability of MNNs with time delays (Guo, Wang, & Yan, 2013b; Hu & Wang, 2010; Wen, Zeng, & Huang, 2012; Wu & Zeng, 2012).

On the other hand, neural networks sometimes have finite modes that switch from one to another at different times according to a switching law generated from a switching logic (Tino, Cernansk, & Benukova, 2004). To describe the switching phenomenon in neural networks, the so-called switched neural networks have been proposed and the stability problems have been considered in Tsividis (1989). Much attention has been paid to switched neural networks due to their extensive applications in many fields such as high-speed signal processing and artificial intelligence (Brown, 1989). Generally speaking, it is difficult to find a common Lyapunov functional to guarantee the stability of a switched system for arbitrary switching signals. In order to find a suitable switching signal to ensure the stability of a switched system, many effective methods have been developed to investigate the analysis and synthesis of switched systems with or without time delay, such as dwell time method (Morse, 1996), average dwell time method (Hespanha & Morse, 1999; Lian & Zhang, 2011; Xin & Li, 2015), mode-dependent average dwell time method (Wu, Shi, Su, & Chu, 2011; Zhao, Zhang, Shi, & Liu, 2012).

Motivated by the above discussion, it is interesting to consider exponential stability of switched MNNs with time-varying delays. To the best of our knowledge, there are few results on this topic. Different from most of the existing papers (Guo, Wang, & Yan, 2013a; Guo et al., 2013b; Guo, Wang, & Yan, 2014; Hu & Wang,

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2010; Wen et al., 2012; Wu & Zeng, 2012), we model a memristor as a continuous system, then MNNs are modeled as complicated, but accurate continuous systems. For the sake of its dynamic analysis, we choose the memristances as uncertain parameters, and the continuous systems are simplified as neural networks with uncertain time-varying parameters. As a result, switched MNNs are modeled as switched neural networks with uncertain time-varying parameters in this paper. In order to get delay-derivative-dependent and delay-dependent criteria for the global exponential stability of switched MNNs, the Lyapunov–Krasovskii functional for each subsystem may contain $x^T(t)P_kx(t)$, $\int_{t-\tau(t)}^{t} e^{\beta(s-t)}g^T(x(s))Q_kg(x(s))ds$, and $\int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\beta(s-t)}\dot{x}^T(s)R_k\dot{x}(s)\,dsd\theta$.

The contribution of this paper includes: (1) switched MNNs are modeled as switched neural networks with uncertain time-varying parameters, (2) the running time ratio for a switching signal is developed, which improves the mode-dependent average dwell time method, and (3) based on the multiple Lyapunov–Krasovskii functionals, the free weighting matrix approach and the average dwell time method, delay-dependent criteria for the global exponential stability of switched MNNs are derived in the form of LMIs.

The paper is organized as follows. In Section 2, the problem is formulated for switched MNNs. In Section 3, sufficient conditions for stability of switched MNNs are obtained with average dwell time method and mode-dependent average dwell time method, respectively. Section 4 gives two examples. Conclusions are drawn in Section 5.

2. Preliminaries and problem formulation

In this section, mathematical models of memristors, MNNs and switched MNNs are introduced. The following notations will be used throughout this paper: I denotes the identity matrix with appropriate dimensions; an asterisk $(*)$ represents a term that is induced by symmetry; for a given matrix X, X^T denotes its transpose; $\lambda_1(P)$ and $\lambda_n(P)$ denotes the maximal and minimal eigenvalue of the positive matrix P, respectively.

2.1. Model of MNNs

Based on the simple mathematical model of the HP memristor (Strukov et al., 2008), a complete flux-controlled model of memristor is provided in Wang, Drakakis, Duan, He, and Liao (2012):

$$M(\varphi) = \begin{cases} R_{off}, & \varphi < c_1 \\ \sqrt{2k\varphi + M^2(0)}, & c_1 \le \varphi < c_2 \\ R_{on}, & \varphi \ge c_2 \end{cases} \tag{1}$$

where $M(\varphi)$ denotes the memristance value, φ is the magnetic flux though the memristor, R_{off} and R_{on} ($R_{off} \gg R_{on}$) are the limit values of the memristor resistance, and the constants k, c_1, c_2 are determined by the memristor and its initial condition.

Using memristors to replace resistors in the circuit realization of the connection links of neural networks, it will result in a neural network called memristive neural network. By Kirchoff's current law, the equations of the ith neuronal states are written as follows:

$$\begin{cases} \dot{\phi}_{1ij}(t) = f_j(x_j(t)) - x_i(t), \quad j = 1, 2, \ldots, n, \\ \dot{\phi}_{2ij}(t) = g_j(x_j(t-\tau(t))) - x_i(t), \quad j = 1, 2, \ldots, n, \\ C_i\dot{x}_i(t) = -\left[\sum_{j=1}^{n}\left(\frac{1}{M_{1ij}(\phi_{1ij}(t))} + \frac{1}{M_{2ij}(\phi_{2ij}(t))}\right) + \frac{1}{R_i}\right]x_i(t) \\ \qquad + \sum_{j=1}^{n}\frac{sign_{ij}f_j(x_j(t))}{M_{1ij}(\phi_{1ij})} + \sum_{j=1}^{n}\frac{sign_{ij}g_j(x_j(t-\tau(t)))}{M_{2ij}(\phi_{2ij})}, \end{cases} \tag{2}$$

where f_j, g_j are the activation functions, $\tau(t)$ is the delay, $x_i(t)$ is the voltage of the capacitance C_i, M_{1ij} is the memristance between the feedback function $f_j(x_j(t))$ and $x_i(t)$, M_{2ij} is the memristance between the feedback function $g_j(x_j(t-\tau(t)))$ and $x_i(t)$, ϕ_{1ij} and ϕ_{2ij} are the fluxes though memristor M_{1ij} and M_{2ij} respectively, and $sign_{ij} = \begin{cases} -1, & i = j; \\ 1, & i \neq j, \end{cases}$ is the sign function.

Since the memristance M_{1ij} and M_{2ij} are bounded due to (1), to reduce the complexity of systems (2), we choose the memristance M_{1ij} and M_{2ij} as uncertain parameters, and view systems (2) as uncertain continuous systems:

$$C_i\dot{x}_i(t) = -\left[\sum_{j=1}^{n}\left(\frac{1}{M_{1ij}(t)} + \frac{1}{M_{2ij}(t)}\right) + \frac{1}{R_i}\right]x_i(t) + \sum_{j=1}^{n}\frac{sign_{ij}f_j(x_j(t))}{M_{1ij}(t)} + \sum_{j=1}^{n}\frac{sign_{ij}g_j(x_j(t-\tau(t)))}{M_{2ij}(t)} \tag{3}$$

where $M_{1ij}(t) \in co\{R_{on}, R_{off}\}$ and $M_{2ij}(t) \in co\{R_{on}, R_{off}\}$.

Denote

$$\begin{aligned} & W_{1ij}(t) = \frac{1}{M_{1ij}(t)}, \qquad W_{2ij}(t) = \frac{1}{M_{2ij}(t)}, \\ & d_i(t) = \frac{1}{C_i}\left[\sum_{j=1}^{n}(W_{1ij}(t) + W_{2ij}(t)) + \frac{1}{R_i}\right], \\ & a_{ij}(t) = \frac{sign_{ij}}{C_i}W_{1ij}(t), \\ & b_{ij}(t) = \frac{sign_{ij}}{C_i}W_{2ij}(t), \\ & A(t) = (a_{ij}(t))_{n\times n}, \qquad B(t) = (b_{ij}(t))_{n\times n}, \\ & D(t) = diag\{d_1(t), d_2(t), \ldots, d_n(t)\}, \\ & x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T, \\ & f(x) = [f_1(x_1), f_2(x_2), \ldots, f_n(x_n)]^T, \\ & g(x) = [g_1(x_1), g_2(x_2), \ldots, g_n(x_n)]^T, \end{aligned} \tag{4}$$

then the overall system (3) can be rewritten as

$$\dot{x}(t) = -D(t)x(t) + A(t)f(x(t)) + B(t)g(x(t-\tau(t))) \tag{5}$$

where $W_{1ij}(t) \in co\{\frac{1}{R_{off}}, \frac{1}{R_{on}}\}$ and $W_{2ij}(t) \in co\{\frac{1}{R_{off}}, \frac{1}{R_{on}}\}$, $i, j = 1, 2, \ldots, n$.

Since all the elements of $D(t)$, $A(t)$ and $B(t)$ are linear with regard to $W_{1ij}(t)$ and $W_{2ij}(t)$, then uncertain conditions of system (5) can be written as follows

$$[D(t), A(t), B(t)] \in co\Omega, \tag{6}$$

where

$$\begin{aligned} \Omega &= \{[D^i, A^i, B^i] | i = 1, 2, \ldots, 2^{2n^2}\} \\ &= \left\{[D(t), A(t), B(t)] | W_{1ij}(t), W_{2ij}(t) \in \left\{\frac{1}{R_{off}}, \frac{1}{R_{on}}\right\}, \right. \\ &\qquad \left. i, j = 1, 2, \ldots, n\right\}. \end{aligned} \tag{7}$$

Remark 1. By choosing the memristances of MNNs (2) as the parameters, we convert the complex nonlinear systems (2) into relative simple nonlinear system (5) with uncertain time-varying parameters. We can use Lyapunov theory and LMI technique to analysis the dynamical properties of MNNs such as stability, passivity and synchronization. Note that the memristive model (1) is continuous, therefore MNN model (5) is also continuous. However, Hu and Wang (Hu & Wang, 2010) viewed the memristor