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# A recurrent neural network for adaptive beamforming and array correction

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ABSTRACT

In this paper, a recurrent neural network (RNN) is proposed for solving adaptive beamforming problem. In order to minimize sidelobe interference, the problem is described as a convex optimization problem based on linear array model. RNN is designed to optimize system's weight values in the feasible region which is derived from arrays' state and plane wave's information. The new algorithm is proven to be stable and converge to optimal solution in the sense of Lyapunov. So as to verify new algorithm's performance, we apply it to beamforming under array mismatch situation. Comparing with other optimization algorithms, simulations suggest that RNN has strong ability to search for exact solutions under the condition of large scale constraints.



## 1. Introduction

Optimization theory is a commonly-used method in engineering area (Corsini & Leoreanu, 2003; Hudson, 1981; Lebret & Boyd, 1997; Li, Yu, Yu, Chen, & Wang, in press; Li, Yu, Yu, Huang, & Liu, 2015; Monzingo & Miller, 1980; Razavizadeh, Ahn, & Lee, 2014; Yu, Ser, Er, Gu, & Li, 2009). In wireless communications, antenna arrays provide an efficient means to detect and process singles arriving from different directions. Recently, the theoretical and applied aspects of beamforming have received great research interests during the last decades (Corsini & Leoreanu, 2003; Lebret & Boyd, 1997; Yu et al., 2009). Compared with a single antenna that is limited in directivity and bandwidth, an array of sensors can arbitrarily change its beampattern through altering array's weight. The fifth generation of mobile communications (5G) also considers it as the key technology (Razavizadeh et al., 2014).

In particular, beamforming is designed to optimize according to some specific criteria, such as minimum variance, maximum entropy, and maximum signal-to-interference-plus-noise ratio (Hudson, 1981; Monzingo & Miller, 1980). Different criteria produce different models such as quadratic programming, second order cone programming and some nonconvex programming. Every model has its own advantages and disadvantages.

Because of large scale of sensors in reality, the need for real-time processing of signals is urgent. Based on the previous work of Tank and Hopfield (1986), there has been increasing achievements in the theory, methodology and application of recurrent neural network for optimization. In 1988, the dynamical canonical nonlinear programming circuit (NPC) was introduced by Kennedy and Chua for nonlinear programming by utilizing a finite penalty parameter which only obtain the approximate optimal (Kennedy & Chua, 1988). Latter on, the Lagrangian network (based on Lagrangian method) was proposed by Zhang and Constantinides for solving convex nonlinear programming problem (Zhang & Constantinides, 1992; Zhu, Zhang, & Constantinides, 1992). And then the primal–dual neural network with global stability was proposed for providing the exact solutions for linear and quadratic programming problems (Bouzerdoum & Pattison, 1993; He, Huang, Li, Che, & Dong, 2015; Lillo, Loh, Hui, & Zak, 1993; Maa & Shanblatt, 1992; Tao, Cao, Xue, & Qiao, 2001; Wang, 1993, 1994; Xia, 1996a; Xia & Wang, 2000). The projection neural network developed by Xia and Wang was proposed to efficiently solve many optimization problems and variational inequalities (Gao, 2004; He, Huang, & Yu, in press; He, Li, & Huang, 2014; He, Li, Huang, & Li, 2014; He, Yu, Huang, Li, & Li, 2014; Hu & Wang, 2007; Hu & Zhang, 2009; Liu & Cao, 2010; Liu, Cao, & Xia, 2005; Liu & Wang, 2011; Xia, 1996b; Xia & Wang, 1998). Moreover, in recent years, recurrent neural network has been widely applied in wide range of programming problems (Li, Yu, Huang, Chen, & He, 2015; Liu & Wang, 2015a, 2015b).

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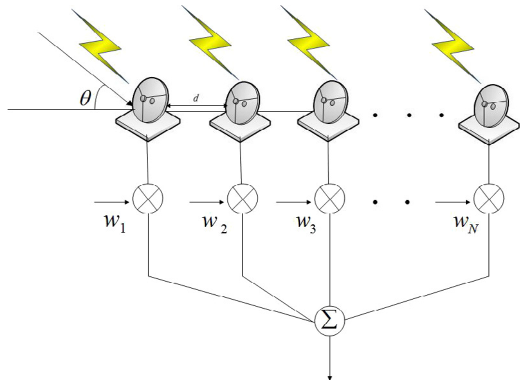


**Fig. 1.** *N* sensors' linear array.

In this paper, we study a adaptive beamforming problem based on linear array model. Firstly, the problem is converted to a convex optimization problem according to arrays' state and plane wave's information. The number of constraints is mainly determined by sampling interval's width, traditional optimization methods are time-consuming and getting poor results. Inspired by the effectiveness and efficiency of neural network optimization method, we adopt this algorithm dealing with beamforming and array mismatch correction. This paper's bright spot is to design a recurrent neural network (RNN) based on beamforming problem's auxiliary cost function and gradient method. Using Lyapunov function theory, it is proven that the proposed neural network is Lyapunov stable and globally convergent to the optimal solution. Comparing with other four classical optimization algorithms, RNN shows superior performance.

The rest of this paper is organized as follows. In the next section, a convex beamforming model is built. In Section 3, a novel recurrent neural network based on gradient method is described. The theoretical analysis of the proposed neural network is presented in Section 4. In Section 5, four comparison algorithms are introduced briefly. Simulation results on the numerical example are given in Section 6. Finally, Section 7 concludes this paper.

## 2. Beamforming model description

In this part, we consider a uniform linear array (ULA) consisting of $N$ isotropic sensors with inter-element spacing $d$ (see Fig. 1). A plane wave of wavelength $\lambda$ is incident on the array from an angle $\theta$, let the first sensor on the left be benchmark sensor, so the array steering vector is $s(\theta) = [1, e^{-j\phi}, \dots, e^{-j(N-1)\phi}]^H$, where $\phi = \frac{2d\pi\cos(\theta)}{\lambda}$. The array response function $G(\theta)$ is given by $G(\theta) = s(\theta)^H \mathbf{w}$, $\mathbf{w} = (w_1, \dots, w_N)^H$ is array weight vector. Actually, we require that the maximum gain of the main lobe is obtained at $\theta_m$ and the maximum sidelobe as small as possible. So the beamforming can be formulated as following Min–Max optimization problem:

**Assumption 1.** Let $\Theta_{tar}$ and $\Theta_{rej}$ be target angle set and reject angle, system output is mainly interfered by non-target angle plane wave.

**Assumption 2.** ALL receiving antennas are omnidirectional antennas and isotropic, transmission signal is narrow-band signal.

**Remark 1.** In fact, we know that antennas have various kind of loss such as medium loss, cable loss and signal reflection loss. In this paper, we regard side lobe as the main interference object. So we consider each antenna as an ideal point source, generally speaking they are isotropic.

**Remark 2.** Generally speaking, $\Theta_{tar}$ always has only one angle, however, $\Theta_{rej}$ always contains many angles. Meanwhile, distance between two sensors is also an influential factor, it is usually set as half of wave's length.

$$\min \max_{\theta_j \in \Omega} |s^H(\theta_j)\mathbf{w}|^2 \quad j = 1, \dots, M \tag{1}$$
$$\text{s.t. } s^H(\theta_m)\mathbf{w} = 1$$
$$\Omega = [-90^\circ, 90^\circ] \setminus \theta_m.$$

In order to convert Min–Max problem to a standard optimization problem form, we add a variable $z$ then model (1) can be rewritten as:

$$\min_w z \tag{2}$$
$$\text{s.t. } |s^H(\theta_j)\mathbf{w}|^2 \leqq z$$
$$s^H(\theta_m)\mathbf{w} = 1.$$

For the sake of expressing the problem from the view of real-value functions and variables, let $s(\theta_j) = rs_j + iIs_j$, $s(\theta_m) = rs_m + iIs_m$, $\mathbf{w} = rw + iIw$, where $rs_j = (rs_{j1}, \dots, rs_{jN})^T$, $Is_j = (Is_{j1}, \dots, Is_{jN})^T$ $j = 1, \dots, M$, $rs_m = (rs_{m1}, \dots, rs_{mN})^T$, $Is_m = (Is_{m1}, \dots, Is_{mN})^T$, $rw = (rw_1, \dots, rw_N)^T$, $Iw = (Iw_1, \dots, Iw_N)^T$, where $i = \sqrt{-1}$.

So model (2) could be reconstructed as a new form:

$$\min e^T x \tag{3}$$
$$\text{s.t. } x^T a_j a_j^T x + x^T b_j b_j^T x - e^T x \leq 0 \quad j = 1, \dots, M$$
$$a_m^T x = 1$$
$$b_m^T x = 0$$
$$e, x, a_j, b_j, a_m, b_m \in \Re^{2N+1}$$
$$e = (0, \dots, 1)^T, \qquad x = (rw_1, \dots, rw_N, Iw_1, \dots, Iw_N, z)^T,$$
$$a_j = (rs_{j1}, \dots, rs_{jN}, -Is_{j1}, \dots, -Is_{jN}, 0)^T,$$
$$b_m = (Is_{m1}, \dots, Is_{mN}, rs_{m1}, \dots, rs_{mN}, 0)^T,$$
$$b_j = (Is_{j1}, \dots, Is_{jN}, rs_{j1}, \dots, rs_{jN}, 0)^T,$$
$$a_m = (rs_{m1}, \dots, rs_{mN}, -Is_{m1}, \dots, -Is_{mN}, 0)^T.$$

The above may be expressed more compactly by defining:

$$A_j = a_j a_j^T + b_j b_j^T \quad B_m = \begin{pmatrix} a_m^T \\ b_m^T \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$A_j \in \Re^{(2N+1)\times(2N+1)}, \qquad B_m \in \Re^{2\times(2N+1)}.$$

So model (3) can be simplified as following expression:

$$\min e^T x \tag{4}$$
$$\text{s.t. } x^T A_j x - e^T x \leq 0 \quad j = 1, \dots, M$$
$$B_m x = b.$$

Obviously, model (4) is a convex optimization problem, it consists of quadratic constraints and linear constraints. Based on model (4), some new models which have practical significance can be evolved:

### 2.1. Power constraint model

We know that, Power consumption is very important in practical applications. In the beamforming signal synthesizer, every signal comes from a receiving sensor amplifying the signal