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Stability and synchronization of memristor-based fractional-order delayed neural networks

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ABSTRACT

Global asymptotic stability and synchronization of a class of fractional-order memristor-based delayed neural networks are investigated. For such problems in integer-order systems, Lyapunov–Krasovskii functional is usually constructed, whereas similar method has not been well developed for fractional-order nonlinear delayed systems. By employing a comparison theorem for a class of fractional-order linear systems with time delay, sufficient condition for global asymptotic stability of fractional memristor-based delayed neural networks is derived. Then, based on linear error feedback control, the synchronization criterion for such neural networks is also presented. Numerical simulations are given to demonstrate the effectiveness of the theoretical results.

1. Introduction

In 1971, circuit theorist Prof. L.O. Chua originally envisioned the existence of a fourth fundamental circuit element called memristor (Chua, 1971). Almost four decades later, the first memristor device was experimentally confirmed by the researchers of Hewlett–Packard (HP) who reported on Nature in 2008 (Strukov, Snider, Stewart, & Williams, 2008). Memristor is a nonlinear resistor with memory, whose the most interesting feature is that it can memorize electric charge flowed in what direction through it in the past. It is with such characteristic which makes it possible to process and storage information with low power. In recent years, it have been showed that memristor devices have many promising applications in pattern recognition, programmable logic, signal processing, reconfigurable computing, brain–computer interfaces, control system and so on (see Driscoll, Quinn, & Klein, 2010; Shin, Kim, & Kang, 2011; Yan, Choe, Nam, Hu, Das, Klemic, Ellenbogen, & Lieber, 2011; Yang, Strukov, & Stewart, 2013). Particularly necessary to point out, in order to simulate the artificial neural network of human brain better, in large-scale nonlinear analog circuit of neural network, the self feedback connection weights and connection weights implemented by the traditional resistors have been replaced by memristors to form a new model of neural networks, which is known as memristor-based neural networks (MNN) (Thomas, 2013). Now, dynamics analysis and applications of MNN have been attracted increasing attention, (see Adhikari, Yang, Kim, & Chua, 2012; Ebong & Mazumder, 2012; Wang & Shen, 2014; Wen, Bao, Zeng, Chen, & Huang, 2013; Wu & Zeng, 2012; Zhang, Shen, & Sun, 2012; Zhang, Shen, Yin, & Sun, 2013) and references therein. In particular, in view of the potential applications in diverse areas such as secure communication (Zhou, Chen, & Xiang, 2005), information science (Bondarenko, 2005), biological system (Molinari, Leggio, & Thaut, 2007), pattern recognition (Haken, 2006) and so on, synchronization of neural networks has gained considerable attention in recent years (Rakkiyappan, Chandrasekar, Park, & Kwon, 2014), which also include MNN for the special feature of memristor. Some positive and interesting results on synchronization of MNN have been obtained, see (Chandrasekar, Rakkiyappan, Cao, & Lakshmanand, 2014; Guo, Wang, & Yan, 2015; Jiang, Wang, Mei, & Shen, 2015; Li & Cao, 2015; Wang & Shen, 2015; Wen, Zeng, Huang, & Chen, 2013b; Wu, Wen, & Zeng, 2012; Zhang & Shen, 2014).

Fractional calculus is a branch of mathematical analysis, which mainly deals with derivatives and integrals of arbitrary non-integer order. Although it has a long mathematical history, the applications of fractional calculus to physics and engineering are only a recent focus of interest (Caponetto, 2010; Podlubny, 1999; Saptarshi & Indranil, 2012; Valerio, 2012). Since fractional

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derivatives are nonlocal and have weakly singular kernels, compared with integer-order calculus, the major advantage is that they provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. In order to better describe the dynamical behavior of the neurons, such as "infinite-memory", some researchers recently introduced it to neural networks to form fractional-order neural networks (FNN) (Anastasio, 1994; Anastassiou, 2012; Lundstrom, Higgs, Spain, & Fairhall, 2008). Previous studies have confirmed that the incorporation of memory terms (a fractional derivative or integral operator) into neural network models is an important improvement (Kaslika & Sivasundaram, 2012). The main advantage of fractional-order neural networks in comparison with integer-order model lies in two aspects, one is its infinite memory, the other is that fractional-order parameter enriches the system performance by increasing one degree of freedom. Very recently, many researchers have focused on FNN, some important and interesting results on dynamics behaviors and synchronization of FNN have been achieved in this new field (Chen, Chai, Wu, Ma, & Zhai, 2013; Chen, Qu, Chai, Wu, & Qi, 2013; Huang, Zhao, Wang, & Li, 2012; Song & Cao, 2014; Stamova, 2014; Wang, Yu, & Wen, 2014; Yu, Hu, Jiang, & Fan, 2014), which was also carried out on memristor-based fractional-order neural networks (MFNN), for instance, global Mittag-Leffler stability and synchronization of MFNN were investigated in Ref. (Chen, Zeng, & Jiang, 2014). Sufficient conditions for projective synchronization of MFNN have been obtained in Ref. (Bao & Cao, 2015). The problem of the existence, uniqueness and uniform stability of MFNN with two different types of memductance functions was extensively investigated in Ref. (Rakkiyappan, Velmurugan, & Cao, 2015). Finite-time stability of fractional-order complex-valued memristor-based neural networks with time delays was discussed in Ref. (Rakkiyappan et al., 2014).

Note that the existing results (Bao & Cao, 2015; Chen et al., 2014; Rakkiyappan et al., 2014) for stability and synchronization of MFNN are independent of delay. In fact, in circuit implementation of neural networks, due to the finite switching speeds of the amplifiers, time delays are inevitable, it will affect the stability of a network by creating instability, oscillation and chaos phenomena. Hence, it is of foremost importance to discuss the stability of delayed MFNN. Ref. (Rakkiyappan et al., 2015) considered stability of fractional-order complex-valued memristor-based neural networks with time delays, but it focuses on finite-time stability from the non-Lyapunov point of view. On the other hand, in recent years, some results about asymptotic stability and synchronization of integer-order MNN are constantly emerging, which are all dependent on the Lyapunov–Krasovskii stability theory for functional differential equations and the linear matrix inequality (LMI) approach. However, similar tools have not been developed for fractional-order nonlinear delayed systems. These approaches could not be extended to fractional-order MNN easily and that is why there are few works about FMNN. Thus, to find out new ways to cope with the problems is very challenging. To the best of our knowledge, there are few results on the global asymptotic stability and synchronization of delayed MFNN.

Motivated by the above discussions, in this paper, by using comparison theorem and stability theorem for fractional-order linear delayed system, a method for analysis on stability of delayed MFNN is established. On the basis of this, sufficient conditions for global asymptotic stability and synchronization of delayed MFNN are presented. The main contribution of this paper can be described in brief as follows: (1) to investigate stability and synchronization of delayed MFNN by employing a comparison theorem for fractional-order linear systems with time delay; (2) our results is on globally asymptotic stability of FMNN with delay for the first time; (3) compared with the earlier results in the literature, the obtained results are more general and less conservative.

The rest of the paper is organized as follows. Some necessary definitions, lemmas and the delayed MFNN are given in Section 2. Two sufficient conditions ensuring the global asymptotic stability and synchronization of delayed MFNN in Section 3 are presented. Three numerical examples are provided in Section 4.

2. Preliminaries and model description

From the Laplace transform of fractional derivative, it is recognized that the main advantage of the Caputo derivative is that it only requires initial conditions given in terms of integer-order derivatives, representing well-understood features of physical situations and making it more applicable to real world problems. So in the rest of the paper, we deal with the fractional-order neural networks involving Caputo derivative.

Definition 1 (*Podlubny, 1999*)**.** The fractional integral with non-integer order $\alpha > 0$ of function $x(t)$ is defined as follows:

$$D_{t_0,t}^{-\alpha}x(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}x(\tau)d\tau,$$

where $\Gamma(\cdot)$ is the Gamma function, $\Gamma(s) = \int_0^\infty t^{s-1}e^{-t}dt$.

Definition 2 (*Podlubny, 1999*)**.** The Caputo derivative of fractional order α of function $x(t)$ is defined as follows:

$$\begin{aligned} {}_{C}D_{t_0,t}^{\alpha}x(t) &= D_{t_0,t}^{-(n-\alpha)}\frac{d^n}{dt^n}x(t) \\ &= \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-\tau)^{n-\alpha-1}x^{(n)}(\tau)d\tau, \end{aligned}$$

where $n-1 < \alpha < n \in Z^+$.

In the following, the notation D^α is chosen as the Caputo fractional derivative operator ${}_{C}D_{0,t}^{\alpha}$.

In the paper, referring to some relevant works on memristor-based fractional-order neural networks (Bao & Cao, 2015; Chen et al., 2014; Rakkiyappan et al., 2014), a class of memristor-based fractional-order neural networks with delay described by the following equation are considered.

$$\begin{aligned} D^{\alpha}x_i(t) &= -c_ix_i(t) + \sum_{j=1}^{n}a_{ij}(x_i(t))f_j(x_j(t)) \\ &+ \sum_{j=1}^{n}b_{ij}(x_i(t))g_j(x_j(t-\tau)) + I_i, \quad i \in N = \{1,2,\ldots,n\}, \end{aligned} \tag{1}$$

where $x_i(t)$ is the state variable of the ith neuron (the voltage of capacitor), $c_i > 0$ is the self-regulating parameters of the neurons, $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ are memristive connective weights, which denote the neuron interconnection matrix and the delayed neuron interconnection matrix, respectively. $f_j, g_j : R \to R$ are bounded feedback functions with and without time-delay between the jth-dimension of the memristor and $x_i(t)$. I_i represents the external input. Let $C([-\tau, 0], R^n)$ be Banach space of all continuous functions, where time-delay $\tau > 0$. The initial conditions of (1) are given by $x(s) = \varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s))^T \in C([-\tau, 0], R^n)$. For more detailed information, one can refer to Refs. (Adhikari et al., 2012; Ebong & Mazumder, 2012; Wang & Shen, 2014; Wen et al., 2013; Wu & Zeng, 2012; Zhang et al., 2012, 2013). According to the feature of memristor, denote

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, & |x_j(t)| < T_j, \\ \check{a}_{ij}, & |x_j(t)| > T_j, \end{cases}$$

$$b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij}, & |x_j(t)| < T_j, \\ \check{b}_{ij}, & |x_j(t)| > T_j, \end{cases}$$