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Synchronization of neural networks with stochastic perturbation via aperiodically intermittent control

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ABSTRACT

In this paper, the synchronization problem for neural networks with stochastic perturbation is studied with intermittent control via adaptive aperiodicity. Under the framework of stochastic theory and Lyapunov stability method, we develop some techniques of intermittent control with adaptive aperiodicity to achieve the synchronization of a class of neural networks, modeled by stochastic systems. Some effective sufficient conditions are established for the realization of synchronization of the underlying network. Numerical simulations of two examples are provided to illustrate the theoretical results obtained in the paper.

1. Introduction

Complex networks are playing more and more important roles in our today's society. A complex network is characterized by a large set of nodes that are connected through a set of links for the communication and interaction purpose. Many real world phenomena, such as those appeared in telecommunication, biological formation, chemical reaction, neural networks, social organization, the World Wide Web, …etc., can be described or modeled by complex networks. Since the seminal papers that started to discuss the "small-world" and "scale-free" properties (Barabasi & Albert, 1999; Watts & Strogatz, 1998), the study of complex networks is not only becoming one of the main research areas in the network society but also brought a great of attention from researchers of different fields. With the viewpoint of complex dynamical networks, many interesting and important dynamical behaviors, such as synchronization, consensus, self-organization, combinatorial optimization, and spatiotemporal chaos of spiral waves, have been studied (Arenas, Guilera, Kurths, Morenob, & Zhoug, 2008; Guan, Liu, Feng, & Wang, 2010; He, Li, Huang, & Li, 2014; He, Li, Huang, Li, & Huang, 2014; He, Yu, Huang, Li, & Li, 2014; Li, Yu, & Huang, 2014; Liu, Wang, Liang, & Liu, 2009; Lu, Ho, & Wang, 2009; Tang, Wang, Gao, Swift, & Kurths, 2012; Wen, Bao, & Zeng, 2013; Wen, Huang, & Zeng, 2015; Wen, Zeng, & Huang, 2014). Over the past decades, synchronization of large-scale complex networks consisting of coupling dynamical systems has been extensively investigated in various disciplines Huang, Ho, and Cao (2005) and Lu and Chen (2006) (Arenas et al., 2008; Strogatz & Stewart, 1993; Tang et al., 2012; Wu, 2007).

As a special class of complex networks, coupled neural networks have been a hot topic because they have wide applications in a variety of areas. One of the focus topics in the investigation of neural networks is the synchronization of all dynamical nodes in a network, resulting from its important applications such as in image processing, general neural networks, secured communication and network updating. Meanwhile, many control methodology has been developed in order to synchronize neural networks governed by nonlinear systems, such as adaptive control (Zhang & Chen, 2009), fuzzy control (Gao, Feng, & Xi, 2014), impulsive control (Li, Li, & Liao, 2011; Li & Song, 2013; Yang, Cao, & Lu, 2011; Zhang, Tang, Miao, & Du, 2013) and intermittent control (Hu, Yu, & Jiang, 2010).

Intermittent control, which was first introduced to control the nonlinear dynamical systems in Zochowski (2000), has been used for a variety of purposes such as manufacturing, transportational and communication. In the past, the intermittent control was mainly periodically intermittent control (Cai, Liu, Xu, & Shen, 2009; Wang, Feng, Xu, & Zhao, 2013; Xia & Cao, 2009; Yu, Hu, Jiang, & Teng, 2012). In Cai et al. (2009), periodically intermittent control is used for the neural networks with time-varying delays to a desired

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orbit. In Yu et al. (2012), the authors have discussed the exponential lay synchronization for delayed fuzzy cellular neural networks via periodically intermittent control. Meanwhile, uncertainties always exist in the real applications, such as stochastic forces on the physical systems and noisy measurements caused by environment complexity. For instance, signals transmitted between nodes of neural networks are unavoidably subject to stochastic perturbations from environment, which may cause information contained in these signals being lost. Therefore, stochastic perturbations cannot be ignored in general (Huang, Feng, & Cao, 2008; Lu, Ho, & Cao, 2008; Pototsky & Janson, 2009; Wang et al., 2013; Yang & Cao, 2009). In Wang et al. (2013), the exponential synchronization of stochastic perturbed complex networks with time-varying delays via periodically intermittent pinning was studied. In Yang and Cao (2009), stochastic synchronization of coupled neural networks with intermittent control was also investigated.

The requirement of periodicity of intermittent control strategy may not be suitable in reality. For example, the generation of wind power in smart grid relies on the various situations of our real world, which is obviously aperiodically intermittent. Therefore, for the theoretical analysis of real applications, it is more practical to consider the synchronization problem under aperiodically intermittent control strategy. To the best of our knowledge, there have been few results for the study of dynamical behaviors in terms of aperiodically intermittent control strategy. In Liu and Chen (2015), synchronization of nonlinear coupled networks via aperiodically intermittent pinning control was investigated. In this paper, we will study the synchronization of nonlinear coupled complex networks with *stochastic* perturbation via aperiodically intermittent control.

Motivated by above discussions, in this paper, we will investigate the problem of synchronization of neural networks with stochastic perturbation via aperiodically intermittent control. Firstly, we will establish sufficient conditions for nonlinear coupled networks under aperiodically intermittent control to achieve synchronization. Secondly, by virtue of properties of Weiner process and estimation techniques, suitable aperiodically intermittent and adaptive aperiodically intermittent controllers are designed to ensure stochastic synchronization for the coupled complex networks with stochastic perturbations. Synchronization criteria obtained in the paper are simple and verifiable, and hence it is practically useful in applications. The obtained theoretical results will be illustrated by numerical simulations in last section.

The structure of this paper is organized as follows: in Section 2, we will introduce the neural network model with stochastic perturbation in terms of aperiodically intermittent control as well as some notations. Sufficient synchronization conditions with mathematical justifications are presented in Section 3. Two illustrative example are given to demonstrate the effectiveness of the proposed approach in Section 4. The paper ends with concluding remarks in Section 5.

Notations. The following notations will be used throughout this paper. $\lambda_{\max}(\cdot)$ stands for the maximum eigenvalue of a real matrix. $\mathbb{R}^+$ and $\mathbb{R}^n$ represent, respectively, the set of nonnegative real numbers and the n-dimensional Euclidean space. $\mathbb{R}^{n\times n}$ is used for the set of all $n\times n$ real matrices. $\|\cdot\|$ is the standard Euclidean norm in $\mathbb{R}^n$. $A=(a_{ij})_{n\times n}$ stands for an $n\times n$ matrix with entries a_{ij}. The superscript T denotes the transpose of a matrix or a vector. I_n is the $n\times n$ identity matrix.

2. Problem formulation and some preliminaries

We consider a neural network system consisting of N identical nodes that are nonlinear coupling with vector-form stochastic perturbations, which is described by

$$\dot{x}_i(t)=\Bigg[-Cx_i(t)+Bf(x_i(t)) + \sum_{j=1,i\neq j}^{N} a_{ij}\left[\phi_j(x_j(t))-\phi_i(x_i(t))\right]\Bigg]dt+\sigma(x_i(t))d\omega(t) \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in\mathbb{R}^n$ represents the state vector of the ith node of the network; $C=\mathrm{diag}(c_1,c_2,\ldots,c_n)$ with $c_k>0$, $k=1,2,\ldots,n$, denotes the rate with which the kth cell rests its potential to the resting state when isolated from other cells and inputs; $B=[b_{ij}]_{n\times n}\in\mathbb{R}^{n\times n}$ represent the connection weight matrix; $A=[a_{ij}]_{n\times n}\in\mathbb{R}^{N\times N}$; $f(x_i(t))=[f_1(x_i(t)),f_2(x_i(t)),\ldots,f_n(x_i(t))]^T$ is a continuous vector; $\sigma(x_i(t))=\sigma(x_1,x_2,\ldots,x_n)\in\mathbb{R}^{n\times n}$ is the noise intensity matrix and $\omega(t)=(\omega_1(t),\omega_2(t),\ldots,\omega_n(t))^T\in\mathbb{R}^n$ is bounded vector-form Weiner process, satisfying $E\omega_j(t)=0$, $E\omega_i^2=1$, $E\omega_j(t)\omega_j(s)=0(s\neq t)$.

In the case that system (1) reaches synchronization, i.e. $x_1(t)=x_2(t)=\cdots=x_N(t)=s(t)$, by introducing a controller into each individual node, where $s(t)\in\mathbb{R}$ is defined as

$$\dot{s}(t)=[-Cs(t)+Bf(s(t))]dt+\sigma(s(t))d\omega(t) \tag{2}$$

and $s(t)$ can be set to be any desired state: either equilibrium point, or a nontrivial periodic orbit, or even a chaotic orbit.

In order to achieve the synchronization objective, the aperiodically intermittent controllers will be applied to some of its nodes. For the convenience of description, we denote $\phi(x_j(t),x_i(t))=\phi_j(x_j(t))-\phi_i(x_i(t))$. Thus the intermittent controlled network can be formulated as

$$\dot{x}_i(t)=\Bigg[-Cx_i(t)+Bf(x_i(t)) + \sum_{j=1,i\neq j}^{N} a_{ij}\phi(x_j(t),x_i(t))+u_i(t)\Bigg]dt+\sigma(x_i(t))d\omega(t) \tag{3}$$

where $u_i(t)$ $(i=1,2,\ldots,n)$ are the intermittent linear state feedback controller and it is constructed as following

$$u_i(t)=\begin{cases}-\varepsilon_i\phi(x_i(t),s(t)), & t\in[t_i,s_i),\\ 0, & t\in[s_i,t_{i+1}),\ i=0,1,2,\ldots\end{cases} \tag{4}$$

where $\varepsilon_i>0$ represents control gain and $\Xi=\mathrm{diag}(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_N)\in\mathbb{R}^{n\times n}$. The synchronization error is defined to be $e_i(t)=x_i(t)-s(t)$. By the controller expression (4), the error dynamics is governed by

$$\dot{e}_i(t)=\Bigg[-Ce_i(t)+Bg(e_i(t)) + \sum_{j=1,i\neq j}^{N} a_{ij}\phi(x_j(t),x_i(t))+u_i(t)\Bigg]dt+\tilde{\sigma}(e_i(t))d\omega(t) \tag{5}$$

where $g(e_i(t))=f(x_i(t))-f(s(t))$ and $\tilde{\sigma}_i(t)=\sigma(x_i(t))-\sigma(s(t))$.

Assumption 1. For the aperiodically intermittent control strategy, there exist two positive scalar $0<\theta<\omega$, such that, for $i=0,1,2$

$$\begin{cases}\inf\limits_i(s_i-t_i)=\theta>0\\ \sup\limits_i(t_{i+1}-t_i)=\omega<+\infty.\end{cases} \tag{6}$$