Distributed parameter estimation in unreliable sensor networks via broadcast gossip algorithms

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ABSTRACT

In this paper, we present an asynchronous algorithm to estimate the unknown parameter under an unreliable network which allows new sensors to join and old sensors to leave, and can tolerate link failures. Each sensor has access to partially informative measurements when it is awakened. In addition, the proposed algorithm can avoid the interference among messages and effectively reduce the accumulated measurement and quantization errors. Based on the theory of stochastic approximation, we prove that our proposed algorithm almost surely converges to the unknown parameter. Finally, we present a numerical example to assess the performance and the communication cost of the algorithm.

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1. Introduction

Distributed parameter estimation (DPE) is one of the fundamental problems in wireless sensor networks (WSNs), and the first step in a wider range of applications such as event detection and classification. In practice, WSNs may be unreliable (Kar, Moura, & Ramanan, 2012; Pereira, López-Valcarce, & Pagès-Zamora, 2013; Wang, Liao, Huang, & Chen, 2015; Zhang & Zhang, 2012) due to external malicious attacks, energy depletion, incorrect sensing and other factors. A convenient way when dealing with agreement in unreliable networks is to exchange messages by an asynchronous framework. The best known one is gossip-based algorithms (Aysal, Yildiz, Sarwate, & Scaglione, 2009; Boyd, Ghosh, Prabhakar, & Shah, 2006; Dimakis, Kar, Moura, Rabbat, & Scaglione, 2010; Fagnani & Frasca, 2011; Wang, Liao, & Huang, 2013; Wu, Zhang, Hou, & Bai, 2014) combined consensus scheme (Belanovic, Macua, & Zazo, 2012; Liu, Lu, & Chen, 2013) which have attracted considerable recent attention. In addition, limitations on the sensor cost, bandwidth, and energy budget dictate that information transmitted between sensors has to be quantized in practice (Aysal, Coates, & Rabbat, 2008; Cai & Ishii, 2011; Lavaei & Murray, 2012; Xiao & Luo, 2005). However, to the best of the authors' knowledge, few authors have considered the distributed parameter estimation problem for unreliable WSNs by an asynchronous quantization communication framework so far.

To overcome the challenges from unreliable network environments, we modify the broadcast gossip algorithm (Aysal et al., 2009; Fagnani & Frasca, 2011; Wang, Liao et al., 2013) to estimate the unknown parameter associated with incompletely informative samples (Wang, Dong, Shen, & Gao, 2013) with noise measured by local sensors and communicated through quantization channels. In a round, each randomly selected sensor broadcasts its quantized estimate to its one-hop neighbors, then each neighbor processes its measurement and updates its estimate through local information exchange if it receives only one broadcasted message, and the remaining sensors sustain their estimates. It is shown that by iterating this procedure, the proposed algorithm is capable of estimating the unknown parameter with probability one.

In summary, our paper makes two main contributions. The first contribution is to provide a simple broadcast based gossiping algorithm for the DPE problem that is fully asynchronous in spite of noise measurements and estimate updates, which will effectively reduce the accumulated measurement and quantization errors raised by algorithms based on synchronous updates. The second one is that the broadcasting sensors of each round are usually probabilistic in nature and the iteration is updated by local

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exchange based on radio transmission, which guarantee that the algorithm is robust to sensor drifting in and out of the network, message interference and link failures.

The remaining part of this paper is organized as follows. Section 2 introduces the background, concepts and techniques used in this paper. Related work and comparison with this paper are discussed in Section 3. Our main contributions are housed in Section 4, which include the asynchronous DPE algorithm design and convergence analysis. Section 5 shows the numerical example and Section 6 draws conclusions.

2. Background, preliminary and primary motivations

In this section, we provide a simple description of background, concepts and techniques that we use for the purpose of modeling distributed digital communication and estimation in WSNs.

In general, a WSN is composed of a large number of geographically distributed sensor nodes characterized by low power constraint and limited computation and communication capabilities (Akyildiz, Su, Sankarasubramaniam, & Cayirci, 2002). With sensor collaboration, a potentially powerful network can perceive the environment to monitor physical phenomena and events of interest. These applications, such as industrial monitoring, battlefield surveillance, health care and home automation, can be usually characterized by some parameters that are not known a priori and need to be estimated based on essential measurements. To save energy and extend the network lifetime, some approaches as energy-aware, in-network processing, multihop communication, and asynchronous routing techniques are often employed for designing WSNs. Specifically, in this paper, we develop an asynchronous communication scheme to achieve these objectives, in which each sensor always remains dormant (inactive) until it is awakened (active) to broadcast its estimate, receive the message and observe the parameter. In addition, the quantization transmission technique is also introduced in digital communication among sensors for saving bandwidth and reducing latency. In the following, we briefly introduce the underlying graph of the WSNs and the framework of probabilistic quantization.

Network model: We model an unreliable WSN as a time-varying undirected graph $\mathcal{G}(t) = \{\mathcal{V}(t), \mathcal{E}(t)\}$ of order $n(t)$, consisting of a set of nodes $\mathcal{V}(t) = \{1, 2, \ldots, n(t)\}$ and a set of edges $\mathcal{E}(t) \subseteq \mathcal{V}(t) \times \mathcal{V}(t)$. An edge in graph $\mathcal{G}(t)$ is denoted by $e_{ij}(t) = (i, j)$. If there is an edge from node j to node i, then it is said that nodes i and j can communicate with each other reliably. The neighborhood set of node i at time t is denoted by $\mathcal{N}_i(t) = \{j | (i, j) \in \mathcal{E}(t)\}$, and the degree of node i at time t is $s_i(t) = |\mathcal{N}_i(t)|$. As usual, we assume that there is no self-loop in $\mathcal{G}(t)$, $\forall t$. Throughout the paper, we denote $n_{\max}$ and $n_{\min}$ by $\max_t\{n(t)\}$ and $\min_t\{n(t)\}$, respectively.

Quantization scheme: Assume that all sensors are equipped with identical uniform probability quantizers $q(\cdot) : \mathbb{R}^d \rightarrow \mathcal{Q}^d$ applied componentwise. Denote $x_i = (x_{i1}, \ldots, x_{id})^\top$. For $i \in \mathcal{V}(t)$, $j \in \{1, \ldots, d\}$, $x_{ij} \in \mathbb{R}$ is identically uniformly bounded to a finite interval $[-U, U]$. Furthermore, we wish to obtain a quantized message $q[x_{ij}]$ with length ℓ bits, where ℓ represents the quantization precision. Therefore, we have $L = 2^\ell$ quantization points given by the set $\Pi = \{\pi_1, \pi_2, \ldots, \pi_L\}$ where $\pi_1 = -U$ and $\pi_L = U$. The points are uniformly spaced such that $\Delta = \pi_{k+1} - \pi_k$ for $k \in \{1, 2, \ldots, L - 1\}$. It follows that $\Delta = 2U/(2^\ell - 1)$. Now suppose $x_{ij} \in [\pi_k, \pi_{k+1})$, then x_{ij} is quantized in a probabilistic manner

$$\mathbb{P}\{q[x_{ij}] = \pi_{k+1}\} = r \quad \text{and} \quad \mathbb{P}\{q[x_{ij}] = \pi_k\} = 1 - r$$

where $r = (x_{ij} - \pi_k)/\Delta$, we refer the reader to Aysal et al. (2008) to make further relevant comments. It is easy to see that when the variable is exactly equal to a quantization centroid, there is zero probability of choosing another centroid. Therefore, it follows

from Xiao and Luo (2005) that the message $q[x_{ij}]$ is an unbiased representation of x_{ij}, i.e., $\mathbb{E}\{q[x_{ij}]\} = x_{ij}$, and $\mathbb{E}\{(q[x_{ij}] - x_{ij})^2\} \leq U^2/(2^\ell - 1)^2 = \Delta^2/4$.

We denote the quantization noisy by $\xi_i = (\xi_{i1}, \ldots, \xi_{id})^\top$, where

$$\xi_{ij} = q[x_{ij}] - x_{ij},$$

it is then an i.i.d. sequence of uniformly distributed random variables on $[-\Delta/2, \Delta/2]$. As pointed out in Aysal et al. (2008), probabilistic quantization is equivalent to a "dithered quantization" method (Kar et al., 2012). It has been shown by Schuchman that the subtractive dithering process yields error signal values that are statistically independent from each other and the input. We, therefore, conclude that $\{\xi_i(t)\}_{t \geq 0}$ is independent of the estimate sequence $\{x_i(t)\}_{t \geq 0}$, i.e., $\mathbb{E}\{\xi_i(t)\}_{t \geq 0} = 0$ and $\mathbb{E}\{\|\xi_i(t)\|_2^2\} \leq d\Delta^2/4$.

3. Related work and discussions

Recently, the paper (Pereira et al., 2013) proposed a distributed expectation–maximization (EM) algorithm to be implemented in a WSN with a decentralized architecture using diffusion-like strategies. In Kar et al. (2012), the authors designed a consensus-based algorithm for estimating the unknown parameter through linear or nonlinear measurements. Further improvement of this work was presented in Zhang and Zhang (2012) to handle the case: WSNs with randomly switching topologies. Very recently, the authors investigated the same problem (Kar et al., 2012) in uniformly connected WSNs with asynchronous and intermittent observations (Wang et al., 2015). Of note is that all of the above are developed based on the framework of synchronous updates to solve DPE problems facing unreliable environments including data observation failures (Kar et al., 2012; Pereira et al., 2013) and link failures (Kar et al., 2012; Pereira et al., 2013; Wang et al., 2015; Zhang & Zhang, 2012). Unlike the above, this paper investigates a novel algorithm based on both asynchronous updates and asynchronous measurements to overcome challenges from unreliable environments including data failures and link failures even sensor equipment failures. Synchronous algorithms have advantages in convergence time, but will accumulate more quantization noise and measurement noise over time, which greatly affect the computations precision. Also, the algorithms based on synchronous updates (Kar et al., 2012; Pereira et al., 2013; Wang et al., 2015; Zhang & Zhang, 2012) are unstable or fail during periods of even modest disruption.

In comparison, the presented algorithm in this paper has overcome these drawbacks and has the following major advantages. First, our algorithm is fully asynchronous thus can effectively reduce the accumulated measurement and quantization noise, and further enhance the computation precision than algorithms based on synchronous updates (Kar et al., 2012; Pereira et al., 2013; Wang et al., 2015; Zhang & Zhang, 2012). In addition, our algorithm does not require error recovery mechanisms thus can solve gracefully DPE problems over networks with a huge number of nodes. Second, the broadcasting sensors of each round are usually probabilistic in nature and the iteration is updated by local exchange based on radio transmission, which will ensure the algorithm to achieve high stability under stress and disruptions including not only common link failures but also sensor equipment failures due to the limited computational and energy resources.

4. Algorithm design and analysis

In this section, we will describe the asynchronous algorithm designed for addressing DPE problems and analyze its convergence properties.