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Robust fixed-time synchronization of delayed Cohen–Grossberg neural networks ☆

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ABSTRACT

The fixed-time master–slave synchronization of Cohen–Grossberg neural networks with parameter uncertainties and time-varying delays is investigated. Compared with finite-time synchronization where the convergence time relies on the initial synchronization errors, the settling time of fixed-time synchronization can be adjusted to desired values regardless of initial conditions. Novel synchronization control strategy for the slave neural network is proposed. By utilizing the Filippov discontinuous theory and Lyapunov stability theory, some sufficient schemes are provided for selecting the control parameters to ensure synchronization with required convergence time and in the presence of parameter uncertainties. Corresponding criteria for tuning control inputs are also derived for the finite-time synchronization. Finally, two numerical examples are given to illustrate the validity of the theoretical results.

1. Introduction

Since the pioneering work of Cohen and Grossberg (1983), the Cohen–Grossberg neural network has been capturing increasing attention from various fields, such as parallel computation, signal and image processing, nonlinear optimization, pattern recognition, among many others (Hornik, Stinchcombe, & White, 1989; Kosko, 1992). There have been many works focusing on the stability and synchronization analysis of the Cohen–Grossberg neural network (Cao & Liang, 2004; Cao & Song, 2006; Chen & Rong, 2003, 2004; Lu & Chen, 2003; Wang, Liu, Li, & Liu, 2006; Yang, Cao, & Yu, 2014; Yu, Cao, & Chen, 2007).

The synchronization of the master–slave systems has been extensively investigated due to its wide applications in communication security and neuroscience (Kanter, Kinzel, & Kanter, 2002). The synchronization indicates that the states of the slave system converge to those of the master system. The asymptotic synchronization, exponential synchronization are incorporated in the category of the infinite time synchronization (Cao & Lu, 2006; Wu, Shi, Su, & Chu, 2012). However, in many practical applications, the synchronization is required to be realized in some finite-time instead of asymptotically, which leads to the study about finite-time synchronization control (Bhat & Bernstein, 2000). Compared with infinite-time synchronization, the finite-time synchronization intrinsically requires a faster convergence speed, and more significantly, the states of the master system and the slave system remain completely identical after some finite time, which is called the settling time (Cao, Ren, & Meng, 2010; Guan, Sun, Wang, & Li, 2012; Liu, Ho, Yu, & Cao, 2014; Yang & Cao, 2010).

Noting that a critical issue about the finite-time synchronization is that the settling time is dependent on the initial conditions of the master–slave systems. Different initial conditions may result in different convergence time. Nevertheless, the initial conditions of many practical systems can hardly be adjusted or even impossible to be estimated, which leads to the inaccessibility of the final settling time and deteriorating of the systems' performance. To overcome this difficulty, Polyakov (2012) proposed a nonlinear feedback design for the fixed-time stabilization of linear control systems, where the definition of fixed-time stable is firstly introduced. Further investigations of fixed-time consensus and stabilization problems have been presented in Parsegov, Polyakov, and Shcherbakov (2013) and Polyakov, Efimov, and Perruquetti (2015).

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In practical situations, due to various perturbations in electric implementations, parameter uncertainties are inevitable. Moreover, in the synchronization problem, parameters of the master system and those of the slave system are not always identical since the two systems may be influenced by different outer disturbances Zhou, Chen, and Xiang (2006) and Zhang, Ma, Huang, and Wang (2010). Compared with some previous works with the assumption that the parametric uncertainties are always identical Ma, Zhang, and Fu (2008), investigations of the case where the connection parameters of the master system and the slave system subject independent uncertainties have more wide applications.

Motivated by the above concerns, this paper investigates the robust fixed-time synchronization control problem for the master–slave Cohen–Grossberg neural networks, which can induce faster convergence independent of the initial conditions and tolerate uncertain parameters. This has not been investigated in the existing literature, which is actually the main contribution of this paper. Different from the previous studies concerning the finite-time synchronization, where the final convergence time is closely related to the initial synchronization errors, the settling time of the fixed-time synchronization can be directly calculated and predesigned regardless of the initial values of the master–slave systems. Additionally, the connection weights of the master system and those of the slave system are assumed to be varying independently in a bounded interval. For the first time, a new control scheme is proposed for the fixed-time synchronization of the master–slave neural networks with parameter uncertainties. The control law involves both continuous terms and discontinuous terms, which together ensure the fixed-time synchronization and eliminate the effect of parameter mismatches. The control protocol fully utilizes the advantages of nonlinear ones, with which faster convergence speed over linear ones can be obtained (Liu, Chen, & Lu, 2009). With the employment of a novel transformation of the Cohen–Grossberg neural networks, simple but efficient Lyapunov functions can be constructed. By virtue of differential inclusion theory (Filippov & Arscott, 1988) and nonsmooth analysis (Clarke, 1998; Wen, Duan, Zhao, Yu, & Cao, 2014), criteria for selecting the control parameters are further derived. Through properly tuning some specific control parameters, the convergence time can be adjusted in advance to satisfy practical requirements regardless of initial synchronization errors.

The rest of the paper is outlined as follows. Model description and some preliminaries concerning the discontinuous theory are presented in Section 2. The control design schemes are proposed and employed to ensure the robust fixed-time synchronization and the robust finite-time synchronization in Section 3. In Section 4, numerical simulations are performed to verify the effectiveness of the main theorems. Finally, conclusions are drawn in Section 5.

2. Model description and preliminaries

In this paper, we consider the following Cohen–Grossberg neural network with time-varying delays

$$\dot{x}_i(t) = -d_i(x_i(t))\left(a_i(x_i(t)) - \sum_{j=1}^{N} b_{ij}f_j(x_j(t)) - \sum_{j=1}^{N} c_{ij}g_j(x_j(t-\tau(t))) - I_i\right), \tag{1}$$

with initial conditions

$$x_{i0}(\theta) = \phi_i(\theta), \quad \theta \in [-\tau, 0],$$

where $i = 1, 2, \ldots, N$, $N \geq 2$ is the number of neurons in the network, $x_i(t)$ represents the state variable of the ith neuron, $d_i(x_i(t))$ denotes the amplification function, $a_i(x_i(t))$ is an appropriately behaved function, matrices $B = (b_{ij}) \in \mathbb{R}^{n\times n}$ and $C = (c_{ij}) \in \mathbb{R}^{n\times n}$ are the connection weight matrix and delayed connection weight matrix, respectively, with elements satisfying $\underline{b}_{ij} \leq b_{ij} \leq \bar{b}_{ij}$, $\underline{c}_{ij} \leq c_{ij} \leq \bar{c}_{ij}$, f_j, g_j denote the activation functions. $\tau(t)$ corresponds to the time varying delays result from the finite speed of the axonal signal transmission and satisfies $0 \leq \tau(t) \leq \tau$, I_i is the external input to the ith neuron.

The master system of the considered master–slave systems is described by (1) while the slave system is formulated as

$$\dot{y}_i(t) = -d_i(y_i(t))\left(a_i(y_i(t)) - \sum_{j=1}^{N} \tilde{b}_{ij}f_j(y_j(t)) - \sum_{j=1}^{N} \tilde{c}_{ij}g_j(y_j(t-\tau(t))) - I_i\right) + u_i(t), \tag{2}$$

with initial conditions

$$y_{i0}(\theta) = \varphi_i(\theta), \quad \theta \in [-\tau, 0],$$

where $\underline{b}_{ij} \leq \tilde{b}_{ij} \leq \bar{b}_{ij}$, $\underline{c}_{ij} \leq \tilde{c}_{ij} \leq \bar{c}_{ij}$, and $u_i(t)$ is the control input to be designed later.

To derive the main results, the following assumptions are introduced.

(A1) $d_i(x)$ is continuous and bounded. Additionally, there exist positive constants $\underline{d}_i$ and $\bar{d}_i$ such that $0 < \underline{d}_i \leq d_i(x) \leq \bar{d}_i$ for $i = 1, 2, \ldots, N$, $x \in \mathbb{R}$.

(A2) The derivative of the amplification function $a_i(x)$ has a positive lower bound, i.e., there exists a positive constant a_i such that $\dot{a}_i(x) \geq a_i > 0$, $x \in \mathbb{R}$.

(A3) The activation functions $f_i(x)$ and $g_i(x)$ are Lipschitz continuous with Lipschitz constants F_i and G_i, respectively, i.e.,

$$|f_i(x) - f_i(y)| \leq F_i|x-y|, \qquad |g_i(x) - g_i(y)| \leq G_i|x-y|, \tag{3}$$

for all $x, y \in \mathbb{R}$.

(A4) The activation functions $f_i(x)$ and $g_i(x)$ are bounded, i.e., there exist $M_i > 0$ such that $|f_i(x)| \leq M_i$ and $|g_i(x)| \leq M_i$, $\forall x \in \mathbb{R}$, $i = 1, 2, \ldots, n$.

Remark 1. Assumption (A4) can be satisfied by many well-known and extensively used activation functions, such as the sigmoid function (Cao & Liang, 2004; Chen & Rong, 2003). Due to the boundedness of activation functions, connection weights' mismatches between the master–slave systems can be regarded as weak heterogeneities to some extent.

Before moving on, choose the transformation function $h_i(x)$ such that

$$\frac{d}{dx}(h_i(x)) = \frac{1}{d_i(x)}, \qquad h_i(0) = 0. \tag{4}$$

According to (A1), $\frac{1}{d_i(x)}$ exists, which is positive and continuous for all $x \in \mathbb{R}$, thus, $h_i(x)$ is a strictly increasing function with respect to x. Letting $z_i(t) = h_i(x_i(t))$, $w_i(t) = h_i(y_i(t))$, it can be obtained directly that $\dot{z}_i(t) = \dot{h}_i(x_i(t))\dot{x}_i(t) = \frac{1}{d_i(x_i(t))}\dot{x}_i(t)$, $\dot{w}_i(t) = \dot{h}_i(y_i(t))\dot{y}_i(t) = \frac{1}{d_i(y_i(t))}\dot{y}_i(t)$, $x_i(t) = h_i^{-1}(z_i(t))$, and $y_i(t) = h_i^{-1}(w_i(t))$. Substituting the above variable transformations into the original master–slave systems (1) and (2), one gets

$$\dot{z}_i(t) = -a_i(h_i^{-1}(z_i(t))) + \sum_{j=1}^{N} b_{ij}f_j(h_j^{-1}(z_i(t))) + \sum_{j=1}^{N} c_{ij}g_j(h_j^{-1}(z_i(t-\tau(t)))) + I_i, \tag{5}$$