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Finite-time synchronization of fractional-order memristor-based neural networks with time delays[☆]

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ABSTRACT

In this paper, we consider the problem of finite-time synchronization of a class of fractional-order memristor-based neural networks (FMNNs) with time delays and investigated it potentially. By using Laplace transform, the generalized Gronwall's inequality, Mittag-Leffler functions and linear feedback control technique, some new sufficient conditions are derived to ensure the finite-time synchronization of addressing FMNNs with fractional order $\alpha : 1 < \alpha < 2$ and $0 < \alpha < 1$. The results from the theory of fractional-order differential equations with discontinuous right-hand sides are used to investigate the problem under consideration. The derived results are extended to some previous related works on memristor-based neural networks. Finally, three numerical examples are presented to show the effectiveness of our proposed theoretical results.

1. Introduction

It is well known that, the fractional calculus is a fast growing field of research in both theoretical and application point of view. It has created an increasing interest and much attention of many researchers, mathematicians and scientist in recent years due to their potential applications in various fields of science and engineering such as physics, continuum mechanics, signal processing, Bioengineering, diffusion wave and electromagnetic (Kilbas, Srivastava, & Trujillo, 2006; Podlubny, 1999). The main feature of fractional order derivatives is nonlocal and has weakly singular kernels, it provides an excellent tool for the description of memory and hereditary properties of various materials and process. Generally, most of the real-world objects are described by the fractional-order differential equations, which shows more accurate results than that of the integer-order counterparts. Moreover, fractional calculus have two main advantages, that is, (i) more degrees of freedom, (ii) infinite memory. Therefore, the study of dynamical analysis of fractional-order differential systems has attracted the interest of many researchers and provided some excellent results in the literature (Ding, Qi, & Wang, 2014; Gao & Liao, 2013; Li, Liao, & Yu, 2003; Lu, 2006).

In the past few decades, neural networks have witnessed as an emerging area of research because of their widespread applications in image processing, signal processing, combinatorial optimization, associative memories, pattern recognition, and so on (Cao & Liang, 2004; He, Liu, Rees, & Wu, 2007). These applications heavily depend on the dynamic behaviors of neural networks. Thus, it is necessary to study the dynamical analysis of neural networks. In Syed Ali (2015a, 2015b) and Syed Ali, Arik, and Saravanakumar (2015), the authors have widely studied the problem of stability, delay-dependent stability and stochastic stability of considered neural networks with Markovian jumping and time-varying delays and presented some good results. Recently, some of the researchers have introduced fractional calculus to neural networks and have formed fractional-order neural networks due to their infinite memory property. In Lundstrom, Higgs, Spain, and Fairhall (2008), the authors pointed out that fractional-order neural network is an effective tool for handling many applications such as parameter estimation due to its memory and hereditary properties. Therefore, it is necessary to investigate the dynamical analysis of fractional-order neural networks and many of the researchers have reported some remarkable results in the literature (Chen, Chai, Wu, Ma, & Zhai, 2013; Kaslik & Sivasundaram, 2012; Wang, Yu, & Wen, 2014; Yu, Hu, & Jiang, 2012).

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On the other hand, memristor-based neural networks is also a hot topic of research in recent years. Memristor is known as fourth basic nonlinear two-terminal circuit device in the electronic circuit theory. It has been firstly proposed theoretically by Chua (1971) and has been realized as the prototype by Hewlett–Packard laboratory (Strukov, Snider, Stewart, & Williams, 2008; Tour & He, 2008). Memristor shared many properties of resistors and the same unit of measurement (Ohm). The main advantage of memristor element is to provide a non-volatile memory storage within a single device structure. According to the memory properties, memristor has been used in many potential applications, such as booting free computers, cell phone battery which would be used for several months (Itoh & Chua, 2009; Wang, Chen, Xi, Li, & Dimitrov, 2009). Moreover, memristor element has been replaced by resistors in the very large scale integrated circuit of neural networks, and then the new model circuit is named as memristor-based neural networks (MNNs). The connection weights of MNNs depend on its state, that is, MNNs is the state-dependent switching system and have more and more complicated properties than neural networks. Therefore, the analysis of MNNs have become more and more noticeable. Most of the researchers and scientist have paid more interest on the analysis of the complicated properties of MNNs and have provided some interesting results in the literature (Chen, Zeng, & Jiang, 2014a; Wen, Zeng, & Huang, 2012; Wu & Zeng, 2012a, 2012b; Zhang, Shen, Yin, & Sun, 2013). In Chen et al. (2014a), the authors have studied the periodic dynamics of MNNs with time-varying delays and some sufficient conditions have been derived to ensure the globally exponentially stable of considered MNNs.

The synchronization of chaotic systems and chaotic neural networks have become an active area of research in nonlinear science. Many authors focused their interest and paid considerable attention to the analysis of synchronization of chaotic neural networks due to their valuable applications in combinational optimization, associative memory, secure communication, pattern recognitions and so on (Milanovic & Zaghloul, 1996; Tan & Ali, 2001). Till date, there were several types of synchronization of chaotic neural networks have been considered in the literature such as complete synchronization, phase synchronization, projective synchronization, lag synchronization and so on (Erjaee & Momani, 2008; Yan & Li, 2007; Yu, Hu, Jiang, & Fan, 2014). The above mentioned types of synchronization are shows that the trajectories of the response system can reach the trajectories of deriving system over the infinite horizon. In the application point of view, the synchronization should be realized in finite-time which is more and more important. Thus, it is necessary to analyze the finite-time synchronization of chaotic neural networks. Recently, many authors have paid their attention and interest for the analysis of finite-time synchronization of chaotic neural networks and some good results has been reported in the literature (Abdurahman, Jiang, & Teng, 2015; Cui, Fang, Zhang, & Wang, 2014; Cui, Sun, Fang, Xu, & Zhao, 2014; Hu, Yu, & Jiang, 2014; Huang, Li, Huang, & He, 2014; Liu, Ho, Yu, & Cao, 2014; Liu, Yu, Cao, & Alsaadi, 2015; Shen, Park, & Wu, 2014; Wu, Cao, Alofi, Abdullah, & Elaiw, 2015; Yang, 2014; Zhang & Feng, 2011). In Yang (2014), the authors studied the finite-time synchronization of neural networks with arbitrary time delays. Some new sufficient conditions have been derived to realize local and global synchronization in finite time for the addressed neural networks based on two different time-delayed feedback controllers in Hu et al. (2014). In Cui, Sun et al. (2014), the authors have studied the finite-time synchronization problem for a class of complex networks with constant time delay. Recently, the authors (Liu et al., 2015) have studied the finite-time synchronization control (FTSC) of complex networks with discontinuous and continuous node dynamics and many sufficient criteria were given to guarantee FTSC by utilizing the finite-time stability theorem. Some new sufficient conditions ensuring the finite-time synchronization of memristor-based chaotic neural networks have obtained by using analysis technique, finite time stability theorem and adding a suitable feedback controller in Abdurahman et al. (2015).

Motivated by the above discussion, the main contribution of this paper is to analyze the problem of finite-time synchronization of fractional-order memristor-based neural networks with time delays. By using Laplace transform, the generalized Gronwall's inequality, Mittag-Leffler functions and linear feedback control technique, some new sufficient conditions are derived to guarantee the finite-time synchronization of considered neural networks with time delays. Recently, several results on the synchronization of memristor-based neural networks (Chen, Zeng, & Jiang, 2014b; Li & Cao, 2015; Wang, Li, Peng, Xiao, & Yang, 2014; Yang, Cao, & Yu, 2014) have been reported in the literature. In Chen et al. (2014b), the authors have extensively studied the global Mittag-Leffler stability and synchronization of memristor-based fractional-order neural networks. In Li and Cao (2015), the authors have proposed new synchronization criteria for memristor-based neural networks by using adaptive control and feedback control techniques. To the best of our knowledge, the problem of finite-time synchronization of fractional-order memristor-based neural networks with time delays is not yet been investigated in the existing literature.

The rest of the paper is organized as follows. In Section 2, some basic definitions, lemmas and assumptions are proposed. Some new sufficient conditions for finite-time synchronization of fractional-order memristor-based neural networks are derived with fractional-order $1 < \alpha < 2$ as well as fractional-order $0 < \alpha < 1$. In Section 4, three numerical examples are given to show the effectiveness of our results. Finally conclusion of this paper is proposed in Section 5.

2. Preliminaries

In this section, we recall some basics of fractional calculus, definitions, assumptions and lemmas which will be useful in deriving our main results.

Definition 1 (*Podlubny, 1999*)**.** The fractional integral of order α for a function f is defined as

$$I^{\alpha}x(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}x(s)ds, \tag{1}$$

where $t \geq t_0$, $\alpha > 0$ and $\Gamma(\cdot)$ is the gamma function, that is, $\Gamma(\tau) = \int_0^{\infty} t^{\tau-1}e^{-t}dt$.

Definition 2 (*Podlubny, 1999*)**.** Caputo's fractional derivative of order α for a function $x \in C^n([t_0,\infty),\mathbb{R})$ is defined by

$$D^{\alpha}x(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}\frac{x^{(n)}(s)}{(t-s)^{\alpha-n+1}}ds, \tag{2}$$

where $t > t_0$ and n is a positive integer such that $n-1 < \alpha < n$. Moreover, when $0 < \alpha < 1$

$$D^{\alpha}x(t) = \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{x'(s)}{(t-s)^{\alpha}}ds.$$

Definition 3 (*Podlubny, 1999*)**.** Mittag-Leffler function is defined as

$$E_{\alpha}(z) = \sum_{p=0}^{\infty}\frac{z^p}{\Gamma(p\alpha+1)}, \tag{3}$$

where $\alpha > 0$ and $z \in \mathcal{C}$.