Neural Networks 69 (2015) 20–28

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Finite-time synchronization for memristor-based neural networks with time-varying delays

Abdujelil Abdurahman, Haijun Jiang *, Zhidong Teng

College of Mathematics and System Sciences, Xinjiang University, Urumqi, 830046, Xinjiang, PR China

ARTICLE INFO

Article history:
Received 17 November 2014
Received in revised form 10 February 2015
Accepted 24 April 2015
Available online 11 May 2015

Keywords:
Memristor
Finite-time synchronization
Neural network
Time-varying delay

ABSTRACT

Memristive network exhibits state-dependent switching behaviors due to the physical properties of memristor, which is an ideal tool to mimic the functionalities of the human brain. In this paper, finite-time synchronization is considered for a class of memristor-based neural networks with time-varying delays. Based on the theory of differential equations with discontinuous right-hand side, several new sufficient conditions ensuring the finite-time synchronization of memristor-based chaotic neural networks are obtained by using analysis technique, finite time stability theorem and adding a suitable feedback controller. Besides, the upper bounds of the settling time of synchronization are estimated. Finally, a numerical example is given to show the effectiveness and feasibility of the obtained results.

1. Introduction

In 1971, Professor Leon O. Chua first proposed the existence of the fourth ideal electrical circuit element to describe the relationship between electric charge and magnetic flux and named it as a memristor, which is a contraction of memory and resistor (Chua, 1971). Almost 40 years later, the Hewlett–Packard research team successfully fabricated the prototype of memristor and showed that a memristor is a nonlinear circuit element and its value, called memristance or memductance, is not unique (Strukov, Snider, Stewart, & Williams, 2008). This is because memristance depends on the magnitude and polarity of the voltage applied to it and the length of the time that the voltage has been applied. When the voltage is turned off, the memristor remembers its most recent value until next time it is turned on Guo, Wang, and Yan (2013). Because of this feature, broad potential applications of the memristor have been identified (Corinto, Ascoli, & Gilli, 2011; Ebong & Mazumder, 2011; Itoh & Chua, 2009; Pershin & Ventra, 2010). For example, memristor can be useful for low-power computation and storage to store information without the need of using power. In addition, memristor can be used to implement programmable analog circuits (Wen, Zeng, Huang, & Chen, 2013). Another important application of memristor is to construct a new model of neural networks, memristor-based neural networks (MNNs), to emulate the human brain (Jin & Wang, 2010).

It is well known that a neural network can be implemented by circuits such as that the Hopfield neural network model can be implemented in a circuit where the self feedback connection weights are implemented by resistors (Chen, Zeng, & Jiang, 2014b). Therefore, by the same logic, we can build MNNs to emulate the human brain by replacing resistors with memristors in the circuits. Such a model is typically a state-dependent nonlinear switching dynamical system (Wu & Zeng, 2013). It can remember its past dynamical history, store a continuous set of states, and be "plastic" according to the pre-synaptic and post-synaptic neuronal activity (Wu, Wen, & Zeng, 2012). It will open up new possibilities in the understanding of neural processes using memory devices, an important step forward to reproduce complex learning, adaptive and spontaneous behavior with electronic neural networks (Wu et al., 2012).

One of the hot topics in the investigation of neural networks is chaos synchronization due to its successful applications in a variety of fields including secure communication, chemical and biological systems, human heartbeat regulation, information science, image processing, and harmonic oscillation generation (Wen, Bao, Zeng, Chen, & Huang, 2013; Wu & Zeng, 2013). Currently, some chaos control and synchronization for memristor-based chaotic systems have been proposed (Chandrasekar, Rakkiyappan, Cao, & Lakshmanan, 2014; Wang, Li, Peng, Xiao, & Yang, 2014; Wen, Bao et al., 2013; Wu et al., 2012; Wu & Zeng, 2013; Wu, Zeng, Zhu, & Zhang, 2011; Zhang & Shen, 2013, 2014). In Wu and Zeng

* Corresponding author.
E-mail address: jianghaijunxju@163.com (H. Jiang).

http://dx.doi.org/10.1016/j.neunet.2015.04.015
0893-6080/© 2015 Elsevier Ltd. All rights reserved.

(2013), the anti-synchronization control of a class of memristive recurrent neural networks was studied by using differential inclusions theory and Lyapunov functional method. In Wu et al. (2012, 2011), by using differential inclusions theory and Lyapunov functional method, the authors were concerned on the exponential synchronization of a class of memristor-based recurrent neural networks. In Wen, Bao et al. (2013), based on the fuzzy theory and Lyapunov method, the global exponential synchronization of a class of memristor-based recurrent neural networks with time-varying delays was considered. In Zhang and Shen (2013, 2014), by employing the non-smooth analysis approach, the authors investigated the exponential synchronization of memristor-based chaotic neural networks with time-varying delays and general activation functions. In Wang et al. (2014), the synchronization control of memristor-based recurrent neural networks with impulsive perturbations was studied. In Chandrasekar et al. (2014), the synchronization of memristor-based recurrent neural networks with two delay components was investigated by second-order reciprocally convex approach.

However, it is worthy of noting that the most of the existing results mentioned above, have been used to guarantee the asymptotic stability or exponential stability of the synchronization error dynamics. This means that the trajectories of the response system can reach to the trajectories of the drive system over the infinite horizon. In the practical engineering process, however, it is more desirable that the synchronization objective is realized in a finite time rather than merely asymptotically. To achieve faster synchronization in control systems, an effective method is using finite-time control techniques. Finite-time synchronization means the optimality in convergence time (Liu, Ho, Yu, & Cao, 2014; Liu, Park, Jiang, & Cao, 2014). Moreover, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties (Nersesov & Haddad, 2008; Yang & Cao, 2010; Zhang, Feng, & Sun, 2012). More recently, the finite-time stabilization and synchronization problem of complex networks, discontinuous neural networks and some other nonlinear systems were studied and many good results were formulated (Liu, Ho et al., 2014; Liu, Park et al., 2014; Nersesov & Haddad, 2008; Yang & Cao, 2010; Zhang et al., 2012). Nevertheless, till now, there are very few or even no published works on the problem of finite-time synchronization for the chaotic MNNs. Therefore, it is interesting to fill the gap.

Motivated by the above discussions, in this paper, we study the finite-time synchronization for a class of MNNs with time-varying delays. First, based on the finite-time stability theory, two different types of controller are introduced. Then, by applying the analysis technique, differential inclusions theory and Lyapunov functional method, some novel sufficient conditions are proposed to ensure the finite-time synchronization for considered MNNs. Finally, some numerical examples are provided to verify the theoretical results established in this paper. Our main results are obtained based on p-norm. It is believed that our results provide some new guidance for the qualitative analysis of memristive recurrent neural networks. These methods may be applied for analyzing other classes of memristive neural networks or some other complex nonlinear memristive systems.

The rest of the paper is organized as follows. In Section 2, the drive–response systems are introduced. In addition, some assumptions and definitions together with some useful lemmas needed in this paper are presented. Next section is devoted to investigating the finite-time synchronization between two chaotic MNNs with time-varying delays. In Section 4, an example with numerical simulations is given to illustrate the effectiveness of the obtained results. Finally, some general conclusions are drawn in Section 5.

2. Preliminaries

In this paper, we consider a class of MNNs with time-varying delays described by the following equation:

$$\begin{aligned}\dot{x}_i(t) &= -d_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x(t)) f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(x(t)) g_j\left(x_j(t-\tau_j(t))\right) + I_i, \qquad (1)\end{aligned}$$

where $i \in \mathcal{I} \triangleq \{1, 2, \ldots, n\}$, $n \geq 2$ denotes the number of neurons in the neural network; $x_i(t)$ corresponds to the voltage of the capacitor C_i; $f_j(x_j(t))$ and $g_j\left(x_j(t-\tau_j(t))\right)$ are the feedback functions; $\tau_j(t)$ is the time-varying delay along the axon of the jth unit from the ith unit and satisfy $0 \leq \tau_j(t) \leq \tau_j$; I_i denotes the external bias on the ith unit; $d_i > 0$ represents the rate with which the ith neuron will reset its potential to the resting state when disconnected from the network; $a_{ij}(x(t))$ and $b_{ij}(x(t))$ represent memristor-based weights, and

$$a_{ij}(x(t)) = \frac{M_{ij}}{C_i} \times sgn_{ij}, \qquad b_{ij}(x(t)) = \frac{W_{ij}}{C_i} \times sgn_{ij},$$

$$sgn_{ij} = \begin{cases} 1, & i = j, \\ -1, & i \neq j, \end{cases}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$; M_{ij} and W_{ij} denote the memductances of memristors P_{ij} and Q_{ij}, respectively. P_{ij} represents the memristor between the feedback function $f_j(x_j(t))$ and $x_i(t)$, and Q_{ij} represents the memristor between the feedback function $f_j(x_j(t-\tau_j(t)))$ and $x_j(t)$. The interested readers can consult the works (Jin & Wang, 2010; Pershin & Ventra, 2010; Wu et al., 2012) to get more explanation about the construction of MNNs.

As is well known, capacitor C_i is invariant while memductances of memristors M_{ij} and W_{ij}, respond to change in pinched hysteresis loops (Chandrasekar et al., 2014; Wang et al., 2014; Wu et al., 2011). Consequently, $a_{ij}(x(t))$ and $b_{ij}(x(t))$ will change. According to the feature of memristor and the current–voltage characteristics, we apply a general mathematical model of the memristance as follows

$$a_{ij}(x(t)) = \begin{cases} \hat{a}_{ij}, & h_i(x(t)) \leq r_i, \\ \check{a}_{ij}, & h_i(x(t)) > r_i, \end{cases}$$

$$b_{ij}(x(t)) = \begin{cases} \hat{b}_{ij}, & h_i(x(t)) \leq r_i, \\ \check{b}_{ij}, & h_i(x(t)) > r_i, \end{cases}$$

where $h_i : \mathbb{R}^n \mapsto \mathbb{R}$ are threshold level functions, $r_i \in \mathbb{R}$ are threshold level, and $\hat{a}_{ij}, \check{a}_{ij}, \hat{b}_{ij}, \check{b}_{ij}$ are constant numbers.

Remark 1. In Wen, Bao et al. (2013), the special case of the threshold level function $h_i(x) = x_i$ for $i \in \mathcal{I}$ is considered. In Chen et al. (2014b), Wu et al. (2012), Wu and Zeng (2013), Wu et al. (2011) and Zhang and Shen (2013, 2014), the authors concerned the case $h_i(x) = |x_i|$ for $i \in \mathcal{I}$. From this point, we can see that the threshold level function considered in this paper is more general.

The initial conditions associated with system (1) are given by

$$x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0], \qquad (2)$$

where $\tau = \max_{j \in \mathcal{I}}\{\tau_j\}$, $\varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s))^T \in C([-\tau, 0], \mathbb{R}^n)$, which denotes the Banach space of all continuous functions mapping $[-\tau, 0]$ into $\mathbb{R}^n$ with p-norm ($p \geq 1$ is a positive integer) defined by

$$\|\varphi\|_p = \left[\sup_{s \in [-\tau, 0]} \sum_{i=1}^{n} |\varphi_i(s)|^p\right]^{\frac{1}{p}}.$$