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Finite-time boundedness and stabilization of uncertain switched neural networks with time-varying delay

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ABSTRACT

This paper deals with the finite-time boundedness and stabilization problem for a class of switched neural networks with time-varying delay and parametric uncertainties. Based on Lyapunov-like function method and average dwell time technique, some sufficient conditions are derived to guarantee the finite-time boundedness of considered uncertain switched neural networks. Furthermore, the state feedback controller is designed to solve the finite-time stabilization problem. Moreover, the proposed sufficient conditions can be simplified into the form of linear matrix equalities for conveniently using Matlab LMI toolbox. Finally, two numerical examples are given to show the effectiveness of the main results.

1. Introduction

Neural networks (NNs) have received considerable attention for their potential applications in many areas, such as image processing, pattern recognition, associative memories, solving certain optimization problems, and so on (Bishop, 1995; Narendra & Parthasarathy, 1990; Watta, Wang, & Hassoun, 1997). Since the switching speed of information processing and the inherent neuron communication is finite, time delays are always unavoidably existent in neural networks, and their existence may lead to instability or significantly deteriorated performances. Then time delay should be taken into account in neural networks models (Arunkumar, Sakthivel, & Mathiyalagan, 2015; Sakthivel, Vadivel, Mathiyalagan, Arunkumar, & Sivachitra, 2015). On the other hand, it is well known that parametric uncertainty is frequently encountered because of the modeling inaccuracies or the changes in the environment of the model. Therefore, it is necessary to consider the introduction of time delay and parametric uncertainty in neural networks models.

Switched systems, which consist of a set of subsystems and a switched signal, have received a large number of scholars' attention due to their successful applications during the past decades. Considering the switch happened in neural networks, some switched neural networks models were proposed, and the corresponding results were put forward on the stability analysis problem (Huang, Qu, & Li, 2005; Yuan, Cao, & Li, 2006). Moreover, the stability and passivity analyses were considered for switched neural networks with time-varying delay in Hu, Cao, Yang, and Hu (2013), Li and Cao (2007), and the stability analysis was investigated for discrete-time switched neural networks in Arunkumar, Sakthivel, Mathiyalagan, and Anthoni (2012) and Hou, Zong, and Wu (2011). The global exponential stability was concerned for switched stochastic neural networks with time-varying delays in Wu, Tian, and Zhang (2014). The synchronization control was studied for switched neural networks with time delay in Yu, Cao, and Lu (2010). The authors in Yang, Huang, and Zhu (2011) studied global exponential synchronization for a class of switched neural networks with time-varying delays and unbounded distributed delays via impulsive control method. The above discussion shows that it is significant to investigate switched neural networks with time delay and parametric uncertainty.

As a prerequisite of application, stability analysis of neural networks has been extensively studied, e.g. asymptotic stability (Cao & Wang, 2005; Liu, Wang, & Li, 2009; Quan, Huang, Yu, & Zhang, 2014), exponential stability (Wang, Lauria, & Fang, 2007; Zhang, Xu, Zong, & Zou, 2009), absolute stability (Xu, Cao, & Sun, 2008), stochastic stability (Arunkumar, Sakthivel, Mathiyalagan, & Park, 2014; Sakthivel, Raja, & Marshal Anthoni, 2013) and references therein. The concept of stability in most existing literatures places

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emphasis on the classical Lyapunov stability, which is related to an infinite-time interval. However, in many practical applications, the dynamical behavior over a fixed finite time interval is more concerned for a system, e.g. the property that the state does not exceed a certain threshold in a finite time interval with a given bound of the initial condition, which is relevant to the finite-time stability. To date, the concept of finite-time stability has been proposed for several decades (Dorato, 1961; Orlowski, 2006), and extended to finite-time bounded by taking the presence of external disturbances and parametric uncertainties into account (Amato, Ariola, & Dorate, 2001; Shen & Li, 2008). In recent years, many results were reported (Chen & Yang, 2014; Cheng, Zhong, Zhong, Zhu, & Du, 2014; Du, Lin, & Li, 2010; He & Liu, 2013; Liu, Shen, & Zhao, 2013; Zhang, Shi, Nguang, Zhang, & Karimi, 2014; Zhang & Yang, 2014) on finite-time problems. It is worth noticing that several valuable results were proposed for the finite-time problems of neural networks. For instance, the finite-time boundedness stability was investigated for neural networks with parametric uncertainties in Shen and Li (2008), for uncertain neural networks with Markovian jumps in He and Liu (2013), Zhang et al. (2014), and the problem of finite-time state estimation was considered for neural networks with time-varying delays in Cheng et al. (2014). To the best of the authors' knowledge, the finite-time boundedness and stabilization of switched neural networks have not been fully investigated in the literature, especially for the finite-time stabilization of the switched neural networks with a switching sequence. It just is the motivation of this paper.

In this paper, the finite-time boundedness and stabilization are studied for a class of uncertain switched neural networks (USNNs) with time-varying delay. The main contribution of this paper is of threefold: (i) The Lyapunov-like function method and average dwell time technique are developed, and several finite-time boundedness criteria are presented for uncertain switched neural networks; (ii) Based on the obtained finite-time boundedness analysis results, the finite-time stabilizable conditions are proposed for the concerned networks, and the corresponding switched controller design is given simultaneously; (iii) The sufficient conditions in our main results can be converted into linear matrix inequalities easily, then Matlab LMI Toolbox can be applied to check the results conveniently.

The remaining part of this paper is organized as follows. In Section 2, the considered uncertain switched neural networks model is formulated and some preliminaries are given. The finite-time boundedness sufficient conditions are obtained for USNNs in Section 3. Section 4 presents the corresponding finite-time stabilization strategy. Numerical examples are provided to illustrate the validity of the proposed method in Section 5, and some conclusions are made in Section 6.

Notation. Throughout this paper, the superscript "T" stands for the transpose of a matrix. $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimension Euclidean space and set of all $n \times n$ real matrices, respectively. A real symmetric matrix $P > 0 (\geq 0)$ denotes P being a positive definite (positive semi-definite) matrix. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ represent for the maximum and minimum eigenvalues of the matrix P respectively. I denotes the identity matrix of appropriate dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The symmetric terms in a symmetric matrix are denoted by $*$.

2. Problem statement

Consider a class of uncertain neural networks with time-varying delay as follows:

$$\begin{cases} \dot{x}(t) = -A(t)x(t) + W_0(t)f(x(t)) + W_1(t)f(x(t-\tau(t))) \\ \quad + B(t)u(t), \\ x(t) = \varphi(t), \quad t \in [-\tau, 0], \end{cases} \tag{2.1}$$

where $x(\cdot) = [x_1(\cdot), x_2(\cdot), \dots, x_n(\cdot)]^T \in \mathbb{R}^n$ is the neuron state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $f(x(\cdot)) = [f_1(x_1(\cdot)), f_2(x_2(\cdot)), \dots, f_n(x_n(\cdot))]^T \in \mathbb{R}^n$ denotes the neuron activation function. $\tau(t)$ is a time-varying delay function with $0 \leq \tau(t) \leq \tau$ and $\dot{\tau}(t) \leq \mu$, and $\varphi(t)(-\tau \leq t \leq 0)$ is the initial condition. $A(t) = A + \Delta A(t)$, $W_0(t) = W_0 + \Delta W_0(t)$, $W_1(t) = W_1 + \Delta W_1(t)$, $B(t) = B + \Delta B(t)$, in which $A = \mathrm{diag}\{a_1, a_2, \dots, a_n\} \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix; $W_0 \in \mathbb{R}^{n\times n}, W_1 \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$ represent the connection weight matrix, the delayed connection weight matrix and control input matrix, respectively; $\Delta A(t)$, $\Delta W_0(t)$, $\Delta W_1(t)$, $\Delta B(t)$ are time-varying parametric uncertainties and assumed to be of the form:

$$[\Delta A(t)\ \Delta W_0(t)\ \Delta W_1(t)\ \Delta B(t)] = HF(t)[E_1\ E_2\ E_3\ E_4],$$

where H, E_1, E_2, E_3, E_4 are known constant matrices of appropriate dimensions, and $F(t)$ is an unknown time-varying matrix with Lebesgue measurable elements bounded by $F^T(t)F(t) \leq I$.

As in Hu et al. (2013) and Li and Cao (2007), considering the switching phenomenon in neural networks (2.1), we study the following uncertain switched neural networks (USNNs) composed of neural networks (2.1) as the individual subsystems:

$$\begin{cases} \dot{x}(t) = -A_{\sigma(t)}(t)x(t) + W_{0\sigma(t)}(t)f(x(t)) \\ \quad + W_{1\sigma(t)}(t)f(x(t-\tau(t))) + B_{\sigma(t)}(t)u(t), \\ x(t) = \varphi(t), \quad -\tau \leq t \leq 0, \end{cases} \tag{2.2}$$

where $A_{\sigma(t)}(t) = A_{\sigma(t)} + \Delta A_{\sigma(t)}(t)$, $W_{0\sigma(t)}(t) = W_{0\sigma(t)} + \Delta W_{0\sigma(t)}(t)$, $W_{1\sigma(t)}(t) = W_{1\sigma(t)} + \Delta W_{1\sigma(t)}(t)$, $B_{\sigma(t)}(t) = B_{\sigma(t)} + \Delta B_{\sigma(t)}(t)$, and $\sigma(t) : [0, +\infty) \longrightarrow \mathbb{N} = \{1, 2, \dots, N\}$ is a piecewise constant switching signal, e.g. when $\sigma(t) = i \in \mathbb{N}$, it implies that the ith subsystem is activated, where $\mathbb{N}$ is a finite set.

Throughout this paper, we assume that the state of the switched neural networks (2.2) does not jump at the switching instants, that is, the trajectory $x(t)$ is everywhere continuous. Moreover, the switching signal $\sigma(t)$ has finite number of switching on any finite interval time. It is worth pointing out that almost all results for switched systems are based on the continuous of the state and the finite of the switching number on any finite interval time, which is the elementary assumption. For the activation function and the parametric uncertainties in the model (2.2), we make the following assumptions.

Assumption 2.1. The each neuron activation function in the USNNs (2.2) is assumed to satisfy

$$l_j^- \leq \frac{f_j(x) - f_j(y)}{x - y} \leq l_j^+, \quad \forall x, y \in \mathbb{R},\ x \neq y,\ j = 1, 2, \dots, n, \tag{2.3}$$

where l_i^- and l_i^+ are some known constants.

For expression convenience, define $L_1 = \mathrm{diag}\{l_1^- l_1^+, \dots, l_n^- l_n^+\}$, $L_2 = \mathrm{diag}\{\frac{l_1^- + l_1^+}{2}, \dots, \frac{l_n^- + l_n^+}{2}\}$, and $L_3 = \mathrm{diag}\{\max\{|l_1^-|, |l_1^+|\}, \dots, \max\{|l_n^-|, |l_n^+|\}\}$.

Assumption 2.2. For any $i \in \mathbb{N}$, the time-varying parametric uncertainties $\Delta A_i(t)$, $\Delta W_{0i}(t)$, $\Delta W_{1i}(t)$, $\Delta B_i(t)$ are assumed to be of the form:

$$[\Delta A_i(t)\ \Delta W_{0i}(t)\ \Delta W_{1i}(t)\ \Delta B_i(t)] = H_i F_i(t)[E_{1i}\ E_{2i}\ E_{3i}\ E_{4i}], \tag{2.4}$$

in which $H_i, E_{1i}, E_{2i}, E_{3i}, E_{4i}$ are known real constant matrices of appropriate dimensions, and the unknown uncertain matrix $F_i(t)$ satisfies $F_i^T(t)F_i(t) \leq I$.

Remark 1. The neural networks model (2.2) in this note not only involves the switch, time-varying delay, and uncertainties in networks parameters, simultaneously, but its restrictions on activation function also have more relaxed expression. Therefore, compared with some existing literatures (Cheng et al., 2014; Shen & Li, 2008), our model is more general to some extent.