Neural Networks 70 (2015) 81–89

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Matrix measure method for global exponential stability of complex-valued recurrent neural networks with time-varying delays[☆]

Weiqiang Gong[a], Jinling Liang[a,b,*], Jinde Cao[a,c]

[a] Department of Mathematics, Southeast University, Nanjing 210096, China
[b] CSN Research Group, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 9 February 2015
Received in revised form 19 May 2015
Accepted 5 July 2015
Available online 21 July 2015

Keywords:
Complex-valued recurrent neural networks
Matrix measure
Halanay inequality
Exponential stability
Time-varying delay

ABSTRACT

In this paper, based on the matrix measure method and the Halanay inequality, global exponential stability problem is investigated for the complex-valued recurrent neural networks with time-varying delays. Without constructing any Lyapunov functions, several sufficient criteria are obtained to ascertain the global exponential stability of the addressed complex-valued neural networks under different activation functions. Here, the activation functions are no longer assumed to be derivative which is always demanded in relating references. In addition, the obtained results are easy to be verified and implemented in practice. Finally, two examples are given to illustrate the effectiveness of the obtained results.

1. Introduction

In the past few years, the dynamic behaviors such as stability and stabilization of recurrent neural networks (see Ahn, 2014; Ahn, Shi, & Wu, in press) have been widely studied due to their extensive applications in classification of pattern recognition, signal processing, image processing, engineering optimization and associative memory, and other areas, see Chen and Wang (2005), Cao and Wang (2014), Cao, Yuan, and Li (2006), Hu and Wang (2002), Liao, Wang, and Zeng (2005), Wang, Zhang, and Yu (2009) and Zhang, Wang, and Liu (2008) and the references therein. In a large amount of applications, complex signals often occur and the complex-valued recurrent neural networks are preferable. Therefore, there have been increasing research interests in the dynamical behaviors of complex-valued recurrent neural networks, for example, see Goh and Mandic (2004, 2007), Hirose (2006), Jankowski, Lozowski, and Zurada (1996a), Lee (2001), Li, Liao, and Yu (2002) and Zhang, Li, and Huang (2014).

Compared with the real-valued neural networks, the states, connection weights and activation functions of the complex-valued neural networks are all complex-valued. Therefore, there are many differences between the real-valued neural networks and the complex-valued ones. In fact, the complex-valued neural networks have much more complicated properties than the real-valued ones in a lot of aspects and hence make it possible to solve many problems that cannot be solved with their real-valued counterparts. For example, both the XOR problem and the detection of symmetry problem cannot be solved with a single real-valued neuron, however, they can be solved with a single complex-valued neuron with the orthogonal decision boundaries (Jankowski, Lozowski, & Zurada, 1996b). Therefore, it is very important to investigate the dynamical behaviors of the complex-valued neural networks, especially the stability of the complex-valued neural networks. In Zou and Song (2013), the complete stability and boundedness of the complex-valued neural networks with time delay have been studied in order to obtain some conditions to guarantee the complete stability of the considered neural networks by using the method of local inhibition and energy minimization. In Zhou and Zurada (2009), a class of discrete-time recurrent neural networks with complex-valued linear threshold neurons has been discussed, and some conditions have been derived to ascertain the global attractivity, boundedness and complete stability of such networks. In Bohner, Rao, and Sanyal (2011), the activation

☆ This work is supported in part by the National Natural Science Foundation of China under Grant 61174136 and 61272530, the Natural Science Foundation of Jiangsu Province of China under Grant BK20130017, the Programme for New Century Excellent Talents in University under Grant NCET-12-0117, the "333 Project" Foundation of Jiangsu Province and the Fundamental Research Funds for the Central Universities and the Graduate Innovation Program of Jiangsu Province (No. KYLX_0083).

* Corresponding author at: Department of Mathematics, Southeast University, Nanjing 210096, China.
E-mail address: jinlliang@seu.edu.cn (J. Liang).

http://dx.doi.org/10.1016/j.neunet.2015.07.003
0893-6080/

dynamics of the complex-valued neural network on a general time scale have been investigated, and several sufficient criteria are derived to guarantee the existence of a unique equilibrium solution. Moreover, the global exponential stability of the considered networks has been also discussed. In Rao and Murthy (2008), a class of generalized discrete-time complex-valued neural network model has been studied, and a sufficient condition for the global exponential stability has been presented; besides that, the existence of the unique equilibrium pattern has also been discussed. For more works on the stability analysis and dissipativity analysis, we refer to the works in Li, Rakkiyappan, and Velmurugan (2015) and Rakkiyappan, Velmurugan, and Li (2015).

There are various approaches for analyzing the neural networks, such as the Lyapunov function method, the energy function method, the synthesis method and so on (Liu, Fang, & Liu, 2009; Michel, Farrell, & Sun, 1990). In Hu and Wang (2012), based on constructing appropriate Lyapunov function and the M-matrix properties, several sufficient conditions have been presented to ascertain the existence and uniqueness of the equilibrium point, and the global exponential stability and the global asymptotic stability for the complex-valued recurrent neural networks. In Liu et al. (2009), a discrete-time complex-valued neural network model has been studied by using the synthesis method, and a stable criterion about network parameters has been derived. In Kuroe, Yoshid, and Mori (2003), the properties of activation functions have been discussed so as to find complex functions with those properties by using the energy function method. However, to the best knowledge of the authors, there are few results on the exponential stability of the complex-valued recurrent neural networks with time-varying delays by using the matrix measure method (Fang & Sun, 2013, 2014), which forms our first motivation to develop the presented research.

Generally speaking, time delay is likely to be present because of the finite switching speed of amplifiers and occur in signal transmissions in the electronic implementation of neural networks, which may influence the dynamical behaviors, since it can bring oscillation, bifurcation and instability to the neural networks, see Ma, Lu, Wang, and Feng (2008), He, Li, Huang, and Li (2013) and Khajanchi and Banerjee (2014) for examples. Hence, it is necessary to study the dynamical behaviors of the delayed neural networks. Over the past few decades, a great deal of work associated with this area has been done by researchers. In Chen and Song (2013), the global stability of complex-valued neural networks with both leakage time delay and discrete time delay on time scales has been studied by constructing appropriate Lyapunov function, and several sufficient conditions have been obtained to ascertain the global stability of the addressed neural networks. In Hu and Wang (2012), the delayed complex-valued recurrent neural networks with two classes of complex-valued activation functions have been investigated, and several sufficient criteria have been obtained to ascertain the existence of the unique equilibrium, the global asymptotic stability and the global exponential stability of the network.

Inspired by the above discussions, a novel approach is proposed for the global exponential stability of the complex-valued recurrent neural networks with time-varying delays. Under two classes of activation functions, several sufficient conditions are presented to ascertain the global exponential stability of the addressed neural networks by using the matrix measure method and Halanay inequality. The remaining part of the paper is organized as follows. In Section 2, the model of the complex-valued recurrent neural networks with time-varying delays is presented, and some preliminaries are briefly outlined. In Section 3, a novel approach is proposed without constructing any Lyapunov functions, several criteria are obtained to ascertain the global stability of the complex-valued neural networks by utilizing the matrix measure method and the Halanay inequality. In Section 4, two numerical examples are given to show the effectiveness of the acquired conditions. Finally, conclusions are drawn in Section 5.

Notations: The notation used throughout this paper is fairly standard. $\mathbb{C}^n, \mathbb{C}^{m\times n}$ and $\mathbb{R}^{m\times n}$ denote the set of n-dimensional complex vectors, $m \times n$ complex matrices and $m\times n$ real matrices, respectively. Let i be the imaginary unit, i.e. $i = \sqrt{-1}$. The superscript 'T' represents the matrix transposition. $X \geq Y$ (respectively, $X > Y$) means that $X - Y$ is positive semi-definite (respectively, positive definite). P^R and P^I denote, respectively, the real and the imaginary parts of matrix $P \in \mathbb{C}^{m\times n}$. $C([t_0-\tau, t_0], \mathbb{R}^n)$ represents the Banach space of continuous vector-valued functions mapping the internal $[t_0 - \tau, t_0]$ into $\mathbb{R}^n$ with the topology of uniform convergence. For $\varphi \in C([t_0 - \tau, t_0], \mathbb{R}^n)$, define its norm as $\|\varphi\|_p = \sup_{t_0-\tau\leq s\leq t_0} \|\varphi(s)\|_p$, where $\|\varphi(s)\|_p$ means the p-norm of $\varphi(s) \in \mathbb{R}^n$.

2. Problem formulation and some preliminaries

Consider the complex-valued recurrent neural networks with time-varying delays described by the following nonlinear delay differential equations:

$$\dot{u}(t) = -Cu(t) + Af(u(t)) + Bg(u(t-\tau(t))) + L, \quad t \geq t_0 \tag{1}$$

where $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T \in \mathbb{C}^n$ is the state vector of the neural networks with n neurons at time t, $C = \text{diag}\{c_1, c_2, \ldots, c_n\} \in \mathbb{R}^{n\times n}$ with $c_k > 0$ $(k = 1, 2, \ldots, n)$ is the self-feedback connection weight matrix, $A = (a_{kj})_{n\times n} \in \mathbb{C}^{n\times n}$ and $B = (b_{kj})_{n\times n} \in \mathbb{C}^{n\times n}$ are, respectively, the connection weight matrix and the delayed connection weight matrix. $L = (l_1, l_2, \ldots, l_n)^T \in \mathbb{C}^n$ is the external input vector. $f(u(t)) = (f_1(u_1(t)), f_2(u_2(t)), \ldots, f_n(u_n(t)))^T : \mathbb{C}^n \to \mathbb{C}^n$ and $g(u(t - \tau(t))) = (g_1(u_1(t - \tau(t))), g_2(u_2(t - \tau(t))), \ldots, g_n\ (u_n(t - \tau(t))))^T : \mathbb{C}^n \to \mathbb{C}^n$ denote, respectively, the vector-valued activation functions without and with time delays, in which $\tau(t)$ is the transmission delay satisfying $0 \leq \tau(t) \leq \tau$ $(\tau > 0)$, and the nonlinear activation functions are assumed to satisfy the conditions given below:

Assumption 1. Let $\nu = \nu_1 + i\nu_2$ with $\nu_1, \nu_2 \in \mathbb{R}$. $f_k(\nu)$ and $g_k(\nu)$ can be expressed by their real and imaginary parts with

$$f_k(\nu) = f_k^R(\nu_1) + if_k^I(\nu_2), \qquad g_k(\nu) = g_k^R(\nu_1) + ig_k^I(\nu_2)$$

where $k = 1, 2, \ldots, n$, and $f_k^R(\cdot),\ f_k^I(\cdot), g_k^R(\cdot),\ g_k^I(\cdot) : \mathbb{R} \to \mathbb{R}$ satisfying

$$|f_k^R(\nu_1) - f_k^R(\overline{\nu}_1)| \leq r_k|\nu_1 - \overline{\nu}_1|,$$
$$|f_k^I(\nu_2) - f_k^I(\overline{\nu}_2)| \leq s_k|\nu_2 - \overline{\nu}_2|;$$
$$|g_k^R(\nu_1) - g_k^R(\overline{\nu}_1)| \leq m_k|\nu_1 - \overline{\nu}_1|,$$
$$|g_k^I(\nu_2) - g_k^I(\overline{\nu}_2)| \leq q_k|\nu_2 - \overline{\nu}_2|;$$

in which r_k, s_k, m_k and q_k are known constants, and $\nu_1,\ \nu_2,\ \overline{\nu}_1,\ \overline{\nu}_2$ are any numbers in $\mathbb{R}$.

With the above Assumption, if we denote $u(t) = x(t) + iy(t)$ with $x(t), y(t) \in \mathbb{R}^n$, then the complex-valued recurrent neural network (1) can be rewritten as follows:

$$\begin{cases} \dot{x}(t) = -Cx(t) + A^Rf^R(x(t)) - A^If^I(y(t)) \\ \qquad + B^Rg^R(x(t-\tau(t))) - B^Ig^I(y(t-\tau(t))) + L^R, \\ \dot{y}(t) = -Cy(t) + A^If^R(x(t)) + A^Rf^I(y(t)) \\ \qquad + B^Ig^R(x(t-\tau(t))) + B^Rg^I(y(t-\tau(t))) + L^I \end{cases} \tag{2}$$

or in a more compact form

$$\begin{aligned} \dot{\alpha}(t) = {} & -C_1\alpha(t) + A_1f_1(\alpha(t)) + A_2f_2(\alpha(t)) \\ & + B_1g_1(\alpha(t-\tau(t))) + B_2g_2(\alpha(t-\tau(t))) + \zeta, \end{aligned} \tag{3}$$