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Robust stability of stochastic fuzzy delayed neural networks with impulsive time window

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ABSTRACT

The urgent problem of impulsive moments which cannot be determined in advance brings new challenges beyond the conventional impulsive systems theory. In order to solve this problem, the novel concept of impulsive time window is proposed in this paper. And the stability problem of stochastic fuzzy uncertain delayed neural networks with impulsive time window is investigated. By combining the discretized Lyapunov function approach with mathematical induction method, several novel and easy-to-check sufficient conditions concerning the impulsive time window are derived to ensure that the model considered here is exponentially stable in mean square. Numerical simulations are presented to further demonstrate the effectiveness of the proposed stability criterion.

1. Introduction

Since mathematical modeling of physical systems and processes in many areas of engineering often leads to complex nonlinear systems, which brings several difficulties to analysis and synthesis, researchers have been seeking effective methods for the control of nonlinear system. It is well known that there has been a turning point in one of the most effective methods in accordance with the advent of the fuzzy model (Takagi & Sugeno, 1985), which is among all of modes to solve the control of complex nonlinear system. Recently, an army of results have been advanced for the fuzzy model which has received increasing attention research because it can provide an effective solution to the control of plants that are mathematically ill-defined, uncertain and nonlinear (Chen & Zheng, 2013; Ho & Sun, 2007; Huang, Ho, & Lam, 2005; Li, Rakkiyappan, & Balasubramaniam, 2011; Rakkiyappan & Balasubramaniam, 2010; Song & Cao, 2007; Takagi & Sugeno, 1985; Wang, Ho, & Liu, 2004; Wu, Su, Shi, & Qiu, 2011; Zhang, Wang, & Liu, 2008). The main feature of the fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model and to express the overall system by fuzzy "blending" of the local linear system models. To date, fuzzy model has been suggested as an alternative approach to conventional control techniques for complex control systems.

For many applications which have been found in various fields, neural networks have been extensively studied and developed (He, Li, & Huang, 2013; Li, Liao, & Lei, 2013; Wang, Liao, & Huang, 2013a, 2013b; Wen, Bao, Zeng, Chen, & Huang, 2013; Wen, Zeng, & Huang, 2013; Wen, Zeng, Huang, & Chen, 2013; Zeng, Huang, & Zheng, 2010; Zeng, Wang, & Liao, 2003; Zeng & Zheng, 2012). In the real world, neural networks are often subjected to external disturbances. Generally speaking, there are two kinds of disturbances considered: parameter uncertainties and stochastic perturbations. Therefore, it is necessary to consider both parameter uncertainties and stochastic effects on the stability of neural networks (Huang, Li, Duan, & Starzyk, 2012; Lu, Cao, Mahdavi, & Huang, 2012; Wong, Zhang, Yang, & Wu, 2013; Yang, Cao, & Lu, 2012; Yu & Cao, 2007). On the other hand, the states of electronic networks and biological networks are often subjected to instantaneous disturbances and experienced abrupt changes at certain instants, which may be caused by switching phenomenon, frequency changes, or other sudden noise, i.e., they exhibit impulsive effects (Li, 2009; Li, Feng, & Huang, 2008; Li & Song, 2013; Li & Zhang, 2009; Lu, Ho, & Cao, 2010, 2011; Song &

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Zhang, 2008; Zhang, Tang, Fang, & Wu, 2012; Zhang, Tang, Miao, & Du, 2013; Zhang, Tang, Miao, & Fang, 2014; Zhang, Tang, Wu, & Fang, 2013). Moreover, time delays are often encountered in the real world due to finite switching speed of amplifiers. Hence, it is of great importance to both investigate delay and impulsive effects on the stochastic stability of neural networks.

It is widely recognized that there are two kinds of impulsive effects, i.e., stabilizing impulses and destabilizing impulses (Lu et al., 2010). Recently, in Wong et al. (2013), a novel strategy named mixed impulses has been proposed. Hence, the previous results which only concerned the lower bound or upper bound of the impulsive sequences become trivial. Moreover, in many actual control systems, the impulsive moments almost cannot be specified in advance. Therefore, it becomes desirable to discuss the impulsive systems with impulsive time window. To the best of the authors' knowledge, the problem of stochastic stability for fuzzy uncertain delayed neural networks with impulsive time window is still an open issue. It is, therefore, the motivation behind our efforts to bridge this gap by studying stochastic stability of fuzzy neural networks with impulsive time window.

Motivated by the shortcoming of the aforementioned research in this area, in this paper, the problem of stochastic stability for uncertain delayed fuzzy neural networks with impulsive time window is investigated. Based on the discretized Lyapunov function method and mathematic induction method, several stability criteria are derived under which stochastic uncertain delayed fuzzy neural networks with impulsive time window are exponentially stable in mean square. The main contributions of this paper can be listed as follows: (1) the stochastic uncertain delayed neural networks both considered the T–S model and impulsive time window are firstly constructed; (2) a unified framework is established to handle stochastic, parameter uncertainty, impulsive time window and fuzzy rule; (3) some approximation algorithms are proposed to compute the lower and upper bounds of the impulsive time window, respectively. The rest of this paper is arranged as follows. In the next section, the problem to be considered and some needed preliminaries are presented. The main results are derived in Section 3. In Section 4, we present several simulation examples to verify the effectiveness of our theoretical results. Finally, the conclusions are drawn in Section 5.

2. Model description and some preliminaries

In this section, some preliminaries are given including model formulation, lemmas, and definitions.

Consider the following stochastic uncertain delayed neural networks:

$$\begin{cases} dx(t) = [-(C+\Delta C)x(t) + (A+\Delta A)f(x(t)) \\ \qquad + (D+\Delta D)f(x(t-\tau(t)))]dt \\ \qquad + [\Delta W_0 x(t) + \Delta W_1 x(t-\tau(t))]dW(t) \\ x(t) = \phi(t), \quad t\in[-\tau,0] \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector associated with the neurons; $C = \mathrm{diag}(c_1, c_2, \ldots, c_n) > 0$ is a positive diagonal matrix, $A = (a_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ are the connection weight matrices; ΔC, ΔA, ΔD, ΔW_0 and ΔW_1 are time-varying matrices on $\mathbb{R}^{n\times n}$ that denote the parameter uncertainties. $f(x(t)) = (f_1(x(t)), f_2(x(t)), \ldots, f_n(x(t)))^{\mathrm{T}}$ denotes the neuron activation function vector; $\tau(t)$ is the transmission delay that satisfies $0 \le \tau(t) \le \tau$, where τ is a positive scalar. $W(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_n(t)]^{\mathrm{T}}$ is an n-dimension Brown motion.

Taking impulsive time window effects into account, we have the following model:

$$\begin{cases} dx(t) = [-(C+\Delta C)x(t) + (A+\Delta A)f(x(t)) \\ \qquad + (D+\Delta D)f(x(t-\tau(t)))]dt \\ \qquad + [\Delta W_0 x(t) + \Delta W_1 x(t-\tau(t))]dW(t), & t\neq t_k \\ x(t_k^+) = B_k x(t_k^-), & t_k \in \mathcal{D}_k \\ x(t) = \phi(t), & t\in[-\tau,0] \end{cases} \tag{2}$$

where $\mathcal{D}_k$ are the time window of impulsive times t_k, i.e., $\mathcal{D}_k = [d_{\min}^k + t_{k-1}, d_{\max}^k + t_{k-1})$, where $d_{\min}^k$ and $d_{\max}^k$ denote the minimum and maximum residence time, respectively. B_k are impulsive gain at impulsive instants t_k; $x(t_k^+) = \lim_{\sigma\to 0^+} x(t_k+\sigma)$, $x(t_k^-) = \lim_{\sigma\to 0^-} x(t_k+\sigma)$.

In this paper, a general class of stochastic fuzzy uncertain delayed neural networks with impulsive time window, are discussed. As in Takagi and Sugeno (1985), the model of stochastic fuzzy uncertain delayed neural networks with impulsive time window is composed of r plant rules that can be described as follows:

Plant Rule i:

IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_p(t)$ is M_{ip}
THEN

$$\begin{cases} dx(t) = [-(C_i+\Delta C_i)x(t) + (A_i+\Delta A_i)f(x(t)) \\ \qquad + (D_i+\Delta D_i)f(x(t-\tau(t)))]dt \\ \qquad + [\Delta W_i^0 x(t) + \Delta W_i^1 x(t-\tau(t))]dW(t), \quad \text{on } t\neq t_k \\ x(t_k^+) = B_{ik}x(t_k^-), \quad \text{on } t_k\in\mathcal{D}_k \\ x(t) = \phi(t), \quad \text{on } t\in[-\tau,0] \end{cases} \tag{3}$$

where $i = 1, 2, \ldots, r$, $M_{ij} (j = 1, \ldots, p)$ are the fuzzy sets, $z(t) = (z_1(t), z_2(t), \ldots, z_p(t))^{\mathrm{T}}$ is the premise variable vector, r is the number of fuzzy IF-THEN rules. It is known that (3) has a unique global solution on $t \ge 0$ with the initial value $\phi(t) \in \mathfrak{L}([-\tau, 0], \mathbb{R}^n)$.

By the singleton fuzzifier, the product inference engine and the center average defuzzifier, the final output of the fuzzy system (3) is inferred as follows:

$$\begin{cases} dx(t) = \sum_{i=1}^{r} h_i(z(t))\{[-(C_i+\Delta C_i)x(t) \\ \qquad + (A_i+\Delta A_i)f(x(t)) \\ \qquad + (D_i+\Delta D_i)f(x(t-\tau(t)))]dt \\ \qquad + [\Delta W_i^0 x(t) + \Delta W_i^1 x(t-\tau(t))]dW(t)\}, & t\neq t_k \\ x(t_k^+) = \sum_{i=1}^{r} h_i(z(t))\mathfrak{B}_i x(t_k^-), & t_k\in\mathcal{D}_k \end{cases} \tag{4}$$

where

$$h_i = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}, \qquad w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t))$$

and $\mathfrak{B}_i = (B_{i1}, B_{i2}, \ldots, B_{ik})^T$, $M_{ij}(z_j(t))$ denotes the grade of membership of $z_j(t)$ in M_{ij}. Note that

$$\sum_{i=1}^{r} h_i(z(t)) = 1, \quad h_i(z(t)) \ge 0,\ i = 1, 2, \ldots, r.$$

Remark 1. From the second equation of (4), it is obvious that a set of control matrices $\mathfrak{B}_i$ and impulsive time window $\mathcal{D}_k$ are to be designed to guarantee the stochastic exponential stability of model (4) in mean square. The impulsive control strategy considered here has some favorable features which are described as follows: (1) The impulse effects here are dependent on the fuzzy rules, namely, both stabilizing and destabilizing impulses are considered. (2) The impulses occur in a random manner in the impulsive time window. (3) The impulsive effects can be distinct at different impulsive instants. Moreover, it is noted that impulsive control strategy considered here can be directly used to realize the state-dependent impulsive control strategy.

Remark 2. In the following sequel, we will illustrate that the impulses considered here encompass the impulses in the previous