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Finite-time synchronization control of a class of memristor-based recurrent neural networks[☆]

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ABSTRACT

This paper presents a global and local finite-time synchronization control law for memristor neural networks. By utilizing the drive–response concept, differential inclusions theory, and Lyapunov functional method, we establish several sufficient conditions for finite-time synchronization between the master and corresponding slave memristor-based neural network with the designed controller. In comparison with the existing results, the proposed stability conditions are new, and the obtained results extend some previous works on conventional recurrent neural networks. Two numerical examples are provided to illustrate the effective of the design method.

1. Introduction

The first memristor (as a contraction of memory and resistor) was originally theorized by Dr. Chua in 1971 (Chua, 1971). He reasoned that, besides the resistor, capacitor and inductor, there should be the fourth circuit element which is now called the memristor (contraction of memory resistor). Although he showed that such an element has many interesting and valuable features, no much attention was paid to his theory because no one could ever build one until nearly 40 years later. In 2008, a group of scientists from Hewlett–Packard Laboratory announced that they had build a prototype of the memristor (Strukov, Snider, Stewart, & Williams, 2008; Williams, 2008). This new circuit element shares many properties of resistors and shares the same unit of measurement (i.e., ohm). Because of its potential applications in next generation computer and powerful brain-like neural computer, it also has generated unprecedented worldwide interest (see Bao & Zeng, 2013; Chua, 1971; Corinto, Ascoli, & Gilli, 2011; Itoh & Chua, 2008; Strukov et al., 2008; Wu, Wen, & Zeng, 2012; Wu, Zeng, Zhu, & Zhang, 2011; Wu, Zhang, & Zeng, 2011; Zhang, Shen, & Sun, 2012).

As far as we know, the neural networks are very important nonlinear circuit networks because of their wide applications in combinatorial optimization, pattern recognition, signal processing and so on, for reference, see Cao, Huang, and Qu (2005), Cao and Wang (2005), Cao, Yuan, and Li (2006), Chen, Cao, and Huang (2002), Hu and Wang (2002), Huang and Cao (2003), Huang, Cao, and Wang (2002), Jiang and Cao (2006), Long and Xu (2011), Shen and Wang (2008), Song (2008), Wu (2009), Zeng, Huang, and Wang (2005), Zeng and Wang (2006), Zeng, Wang, and Liao (2005) and Zhu and Cao (2011). In Refs., Bao and Zeng (2013), Wu et al. (2012), Wu, Zeng et al. (2011), Wu, Zhang et al. (2011) and Zhang et al. (2012) have studied a new model by using memristor instead of resistors where the connection weights change according to its state, i.e., a state-dependent switching recurrent neural networks. Such applications depend on the stability of networks. Therefore, stability is one main property of networks. According to the work in Cao et al. (2005), Cao and Wang (2005), Cao et al. (2006), Hu and Wang (2002), Huang and Cao (2003), Huang et al. (2002), Shen and Wang (2008), Zeng and Wang (2006) and Zhang et al. (2012), a great many results have been reported about the asymptotical and exponential stability of recurrent neural networks. However, for the purpose of control and supervision, a finite time stability of the error system is often desired, particularly in engineering fields (Shen & Huang, 2009). In general, an asymptotically or exponentially synchronization with the controller cannot guarantee that the system under study achieves the control performance of fast convergence (Wu, Zeng et al., 2011), while a finite-time synchronization with the controller possesses such a property which means the optimality in convergence time. And thus far, there are few published papers considering finite-time synchronization of memristor-based neural networks. Therefore,

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considering the actual needs, it is essential to study the finite-time synchronization of memristor-based neural networks.

Different from the previous works (Wu et al., 2012; Wu, Zeng et al., 2011; Zhang et al., 2012), in this paper, we will mainly deal with the problem of finite-time synchronization for a class of memristor-based recurrent neural networks as follows:

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))g_j(x_j(t)) + I_i,$$
$$t \geq 0,\ i = 1, 2, \ldots, n, \tag{1.1}$$

where

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, & |x_i(t)| \leq T_i, \\ \check{a}_{ij}, & |x_i(t)| > T_i, \end{cases} \tag{1.2}$$

in which switching jumps $T_i > 0$, $\hat{a}_{ij}, \check{a}_{ij}$, $i, j = 1, 2, \ldots, n$, are all constant numbers, $g_j(\cdot) : \Re \to \Re$, $j = 1, 2, \ldots, n$ is continuous function, I_i denotes external bounded input.

Remark 1. The authors in Wu et al. (2012) and Wu, Zeng et al. (2011) have given a clear exposition about the relation between memristances and the coefficients of switching system (1.1), so researchers can consult (Wu et al., 2012; Wu, Zeng et al., 2011) to get more explanation.

The organization of this paper is as follows. Some preliminaries are introduced in Section 2. The main results are given in Section 3. And then, numerical simulations are given to demonstrate the effectiveness of the proposed approach in Section 4. Finally, this paper ends by a conclusion.

2. Model description and preliminaries

In this paper, for convenience, some notations are introduced:

Throughout this paper, solutions of all the systems considered in the following are intended in Filippov's sense (see Filippov, 1988). $[\cdot, \cdot]$ represents the interval. We define $\|\mathbf{v}\| = [\sum_{i=1}^{n} \mathbf{v}_i^2]^{\frac{1}{2}}$, for $\forall \mathbf{v} = (\mathbf{v}_1(s), \mathbf{v}_2(s), \ldots, \mathbf{v}_n(s))^T \in \Re^n$. For matrices $A \in \Re^{n\times m}$, A^T denotes its transpose, $\lambda_{\max}(P)$, $\lambda_{\min}(P)$ respectively represents the maximum and minimum eigenvalue of matrix P, and $\|\mathcal{Q}\|$ denotes the operator norm of matrix $\mathcal{Q}$, i.e., $\|\mathcal{Q}\| = [\lambda_{\max}\mathcal{Q}^T\mathcal{Q}]^{\frac{1}{2}}$. Let $\bar{a}_{ij} = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\underline{a}_{ij} = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, for $i, j = 1, 2, \ldots, n$. For matrix $M = (m_{ij})_{n\times n}$, $H = (h_{ij})_{n\times n}$, $M \gg H(M \ll H)$ means that $m_{ij} \geq h_{ij}(m_{ij} \leq h_{ij})$, for $i, j = 1, 2, \ldots, n$. And by the interval matrix $[M, H]$, it follows that $M \ll H$. For $\forall \mathcal{L} = (l_{ij})_{n\times n} \in [M, H]$, it means $M \ll \mathcal{L} \ll H$, i.e., $m_{ij} \leq l_{ij} \leq h_{ij}$ for $i, j = 1, 2, \ldots, n$.

In addition, the initial conditions of system (1.1) are given by $x(s) = \psi(s) = (\psi_1(s), \psi_2(s), \ldots, \psi_n(s))^T \in \Re^n$.

First, by the theory of differential inclusions, from system (1.1), we have

$$\dot{x}_i(t) \in -x_i(t) + \sum_{j=1}^{n} [\underline{a}_{ij}, \bar{a}_{ij}]g_j(x_j(t)) + I_i,$$
$$t \geq 0,\ i = 1, 2, \ldots, n, \tag{2.1}$$

or equivalently, for $i, j = 1, 2, \ldots, n$, there exist $a_{ij} \in [\underline{a}_{ij}, \bar{a}_{ij}]$, such that

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} a_{ij}g_j(x_j(t)) + I_i,$$
$$t \geq 0,\ i = 1, 2, \ldots, n. \tag{2.2}$$

Consider system (2.2) as the master system and the corresponding slave system as:

$$\dot{y}_i(t) = -y_i(t) + \sum_{j=1}^{n} a_{ij}g_j(y_j(t)) + I_i + u_i(t),$$
$$t \geq 0,\ i = 1, 2, \ldots, n, \tag{2.3}$$

where the initial conditions $y(s) = \varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s))^T \in \Re^n$, and $u_i(t)$ $(i = 1, 2, \ldots, n)$ is the appropriate control input that will be designed in order to obtain a certain control effectiveness. The initial conditions associated with system (2.2) are of the form $y_i(t) = \psi_i(t) \in \Re$, $i = 1, 2, \ldots, n$.

Referring to some relevant works in Wu et al. (2012) and Wu, Zeng et al. (2011), we make the following Assumptions (A1) and (A2):

(A1) The functions g_i, $i = 1, 2, \ldots, n$ are bounded and satisfy the Lipschitz condition with a Lipschitz constant $L_i > 0$, i.e.,

$$|g_i(x) - g_i(y)| \leq L_i|x - y| \quad \text{for all } x, y \in \Re.$$

(A2) For $i, j = 1, 2, \ldots, n$,

$$[\underline{a}_{ij}, \bar{a}_{ij}]g_j(x_j(t)) - [\underline{a}_{ij}, \bar{a}_{ij}]g_j(y_j(t))$$
$$\subseteq [\underline{a}_{ij}, \bar{a}_{ij}](g_j(x_j(t)) - g_j(y_j(t))). \tag{2.4}$$

Let $e(t) = (e_1(t), e_2(t), \ldots, e_n(t))^T$ be the synchronization error, where $e_i(t) = y_i(t) - x_i(t)$, and $f_j(e_j(t)) = g_j(x_j(t)) - g_j(y_j(t))$.

Under the Assumptions (A1) and (A2), applying Lemma 3.1 in Guo and Huang (2009), we know that the each solution $x(t)$ of the system (2.2) with the initial condition exists on the interval $[0, +\infty)$.

Supposed that the Assumption 2 is satisfied, applying the theories of set-valued maps and differential inclusions, we can get the synchronization error differential inclusions as

$$\dot{e}_i(t) \in -e_i(t) + \sum_{j=1}^{n} [\underline{a}_{ij}, \bar{a}_{ij}]f_j(e_j(t)) + u_i(t),$$
$$t \geq 0,\ i = 1, 2, \ldots, n, \tag{2.5}$$

or equivalently, for $i, j = 1, 2, \ldots, n$, there exist $a^*_{ij} \in [\underline{a}_{ij}, \bar{a}_{ij}]$, such that

$$\dot{e}_i(t) = -e_i(t) + \sum_{j=1}^{n} a^*_{ij}f_j(e_j(t)) + u_i(t),$$
$$t \geq 0,\ i = 1, 2, \ldots, n, \tag{2.6}$$

with initial conditions $\phi_i(t) = \psi_i(t) - \varphi_i(t)$, $i = 1, 2, \ldots, n$, where $f_j(e_j(t)) = g_j(y_j(t)) - g_j(x_j(t))$, $i, j = 1, 2, \ldots, n$.

Obviously, $f_j(0) = 0$, $j = 1, 2, \ldots, n$. The controllers u_i, $i = 1, 2, \ldots, n$ are designed to stabilize the zero solution of the system (2.6) with initial condition in finite time.

Definition 1. The zero solution of the system (2.6) is said to be finite-time stable (on an open neighborhood $\mathcal{U} \subset \mathcal{D}$ of the origin) if:

(1) there exists a function $T : \mathcal{U} \setminus \{0\} \to (0, \infty)$, such that $\forall e_0 \in \mathcal{U}$, the solution $\psi(t, e_0)$ of the system (2.6) is defined and $\psi(t, e_0) \in \mathcal{U} \setminus \{0\}$ for $t \in [0, T(e_0))$ and $\lim_{t\to T(e_0)} \psi(t, e_0) = 0$, then, $T(e_0)$ is called the settling time.
(2) for all $\epsilon > 0$, there exists $\delta(\epsilon) > 0$ such that for every $e_0 \in (\mathcal{B}_{\|\cdot\|_{2,n}}(\delta(\epsilon) \setminus \{0\})) \bigcap \mathcal{U}$, $e(t, e_0) \in \mathcal{B}_{\|\cdot\|_{2,n}}(\epsilon)$ for all $t \in [0, T(e_0))$.

When $\mathcal{U} = \mathcal{D} = \Re^n$, the zero solution is said to be globally finite-time stable. Furthermore, if only (1) is fulfilled then the origin of system (2.6) is said to be finite-time attractive.

In order to prove that the error dynamic system (2.6) can be guaranteed to converge to the equilibrium points in finite time, some lemmas should be given firstly as follows:

Lemma 1 (*Yang & Cao, 2010*). *Assume that a continuous, positive-definite function $V : \mathcal{D} \to \Re^+$, real numbers $\alpha > 0$, $0 < \eta < 1$ and*