Neural Networks 63 (2015) 223–233

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Convergence and attractivity of memristor-based cellular neural networks with time delays☆

Sitian Qin [a,b], Jun Wang [c,a,*], Xiaoping Xue [d]

[a] School of Control Science and Engineering, Dalian University of Technology, Dalian 116023, China
[b] Department of Mathematics, Harbin Institute of Technology at Weihai, Weihai, 264209, China
[c] Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, Hong Kong
[d] Department of Mathematics, Harbin Institute of Technology, Harbin 150001, China

ARTICLE INFO

Article history:
Received 24 July 2014
Received in revised form 28 October 2014
Accepted 3 December 2014
Available online 18 December 2014

Keywords:
Memristor
Cellular neural networks
Finite-time convergence
Positive invariance
Attractivity

ABSTRACT

This paper presents theoretical results on the convergence and attractivity of memristor-based cellular neural networks (MCNNs) with time delays. Based on a realistic memristor model, an MCNN is modeled using a differential inclusion. The essential boundedness of its global solutions is proven. The state of MCNNs is further proven to be convergent to a critical-point set located in saturated region of the activation function, when the initial state locates in a saturated region. It is shown that the state convergence time period is finite and can be quantitatively estimated using given parameters. Furthermore, the positive invariance and attractivity of state in non-saturated regions are also proven. The simulation results of several numerical examples are provided to substantiate the results.

1. Introduction

It was over three decades ago that Chua physically conceived the existence and mathematically analyzed the properties of a brand-new two-terminal circuit element called the memristor coined as a contraction of memory and resistor (see Chua, 1971). Chua believed that memristor has every right to be the fourth passive circuit element along with the three classical ones (i.e., the resistor, inductor, and capacitor) already in existence for centuries. Unlike other two-terminal devices, the value of memristor (i.e., memristance) does not depend on the instantaneous inputs alone, but depends on how signals are applied over time and hence they exhibit memory effects. For example, when the voltage is turned off, the memristor remembers its most recent value until next time when it is turned on. Although the memristor was extensively investigated in academia in the 1970s, its physical device was not discovered until 37 years later. In 2008, Williams and his group at Hewlett–Packard Laboratories in Strukov, Snider, Stewart, and Williams (2008) announced that they built a solid-state memristor, which was modeled as a thin semiconductor film (TiO_2) sandwiched between two metal contacts. After it, the memristor became more popular ever since due to its unprecedented behavior and potential applications in next generation computers and powerful brain-like neural computers (see Chua, 2011, Fouda & Radwan, 2013, Kim, Sah, Yang, Roska, & Chua, 2012, Lu, 2012, Itoh & Chua, 2009, Talukdar, Radwan, & Salama, 2012, Thomas, 2013).

A memristor works like a biological synapse, with its memristance varying with experience, or with the current flowing through it over time (see, for instance, Anthes, 2011). This special behavior can be used in artificial neural networks, such as pattern recognition or signal processing from sensor arrays, in a way that mimics the human brain (see Anthes, 2011). The cellular neural networks (CNNs), introduced by Chua and Yang in Chua and Yang (1988a, 1988b), are nonlinear dynamic circuits consisting of many processing units called cells in two-dimensional array. Besides their diversified applications for signal processing, images processing, and pattern recognition, the dynamic behaviors of CNNs are theoretically analyzed since last decade (see Cao, Wang, & Liao, 2003, Liao & Wang, 2003, Lu, Wang, & Chen, 2011, Wang, Lu, & Chen, 2010, Zeng & Wang, 2006, 2008, 2009, Zeng, Wang, & Liao, 2004). The inter-neuron connections of conventional CNNs are implemented by using resistors, which do not have memory function. Compared

☆ The work described in the paper was supported by the Research Grants Council of the Hong Kong Special Administrative Region (China, Project no. CUHK416811E) and China Postdoctoral Science Foundation (China, Project No. 2013M530915).

* Corresponding author at: Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, Hong Kong.
E-mail addresses: qinsitian@163.com (S. Qin), jwang@mae.cuhk.edu.hk (J. Wang), xiaopingxue@263.net (X. Xue).

http://dx.doi.org/10.1016/j.neunet.2014.12.002
0893-6080/

with resistor, memristor is more suitably used as synaptic connections in CNNs and memristor-based cellular neural networks (MCNNs) would be more powerful. It was shown in Cantley, Subramaniam, Stiegler, Chapman, and Vogel (2011) and Pershin and Di Ventra (2010) that MCNNs have great vitality and advantages as a more efficient approach for further development of neural network implementations.

It is well known that successful applications of recurrent neural networks (RNNs) rely largely on their dynamical properties, such as stability, periodic oscillatory, chaos, and bifurcation. In 2010, based on theory of Filippov (1964), authors in Hu and Wang (2010) spearheaded the dynamic analysis of memristor-based recurrent neural networks (MRNNs) and studied global uniform asymptotic stability by constructing proper Lyapunov functional. Since then, dynamical analyses of MRNNs have received considerable attention (see Bao & Zeng, 2013, Guo et al. (2013, in press-a,b), Wang & Shen, 2013, Wen & Zeng, 2012, Wen, Zeng, & Huang, 2012, 2013, Wu & Zeng, 2012, 2013, Wu, Zhang, & Zeng, 2011, Zhang, Shen, Quan, & Sun, 2012, 2013). In most of the works, the mathematical model of MRNNs is expressed as an implicit differential equation with discontinuous right side. In order to analyze the class of discontinuous differential equations, most works adopt the theory introduced in Filippov (1964). This theory has become a standard mathematical tool in MRNNs focus on the stability of equilibrium (or periodic solution).

In most existing works, the main focus is on the existence of a unique equilibrium and its stability. However, in practical applications, the neural network is desired to have many attractors. Neural networks with multiple attractors are very useful in practice, such as pattern recognition and associative memory (see Chang, Kuang, & Shih, 2006, Nie & Cao, 2011, Wang et al., 2010). Recently, it is shown in Guo, Wang, and Yan (in press-a) that the number of equilibria of an n-neuron MCNN is up to 2^{2n^2+n} in contrast to 2^n in a conventional CNN (i.e., 2^{2n^2} times more). Hence, it is deemed both necessary and desirable to study local convergence and attractivity of MCNNs.

The rest of this paper is structured as follows. In Section 2, we introduce an MCNN model and some related preliminaries. In Section 3, we prove the existence and essential boundedness of global solution of the MCNN model, and then characterize finite-time convergence and positive invariance of the MCNN in different regions of its state space. In Section 4, we discuss simulation results of several numerical examples to substantiate the effectiveness of the results. Finally, conclusions are made in Section 5.

2. Preliminaries

In this section, a memristor model and an MCNN model are introduced along with definitions and lemmas.

2.1. Memristor model

Memristor is a two-terminal passive device characterized by a constitutive relation (i.e., memristance) between two mathematical variables q and φ, representing the time integral of the element's current $i(t)$, and voltage $v(t)$; namely,

$$q(t) = \int_{-\infty}^{t} i(\tau)d\tau, \qquad \varphi(t) = \int_{-\infty}^{t} v(\tau)d\tau. \tag{1}$$

More precisely, the memristance M of a memristor can be expressed as

$$M = \frac{d\varphi}{dq} = \frac{v(t)}{i(t)}. \tag{2}$$

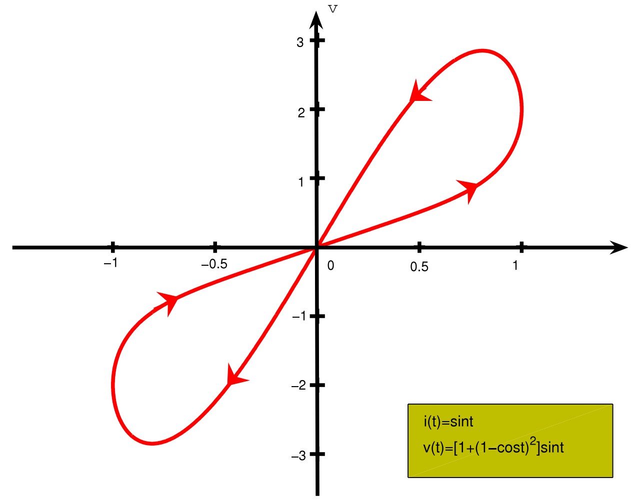

Fig. 1. Typical i–v characteristic of memristor: double-valued Lissajous figure of $(v(t), i(t))$ for all times t except when it passes through the origin, where the loop is pinched.

It is easy to find that if voltage $v(t)$ is a linear function of current $i(t)$, then memristance M is a constant, that is, the memristor degenerates to a resistor.

Fig. 1 depicts the typical i–v characteristic of the memristor (see Chua, 2011 for more details), which shows that the memristance of the device depends on the voltage applied and direction of current flow. Similar to the piecewise linear model in Hu and Wang (2010), Wen et al. (2012) and Wu and Zeng (2012), a simplified mathematical model of memristance can be defined as follows:

$$M(\dot{v}(t)) = \begin{cases} M', & \text{if } \dot{v}(t) < 0 \\ M'', & \text{if } \dot{v}(t) > 0 \\ \lim\limits_{s\to t^-} M(\dot{v}(s)), & \text{if } \dot{v}(t) = 0 \end{cases} \tag{3}$$

where M' and M'' are known constants relating to memristance, and $\lim_{s\to t^-} M(\dot{v}(s))$ means that the memristance keeps its previous value. Obviously, the memristor is a switchable device and its memristance may be discontinuous.

2.2. Memristor-based CNN

The memristor-based cellular neural network (MCNN) is a cellular neural network whose connections are implemented by using memristors. A mathematical model of the MCNN can be expressed as follows:

$$\dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij}(f_j(x_j(t)) - x_i(t)) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(f_j(x_j(t-\tau)) - x_i(t-\tau)) f_j(x_j(t-\tau)) + u_i, \tag{4}$$

where for $i = 1, \ldots, n$; $x_i(t)$ is the state of the ith neuron at time t, d_i denotes the self-inhibition rate of the ith neuron, $a_{ij}(\cdot)$ and $b_{ij}(\cdot)$ are respectively the feedback connection weights implemented by using memristors with and without time delay; u_i is the ith external input, τ is a time delay, and f_i is a linear saturation activation function defined as

$$f_i(s) = \begin{cases} 1, & \text{if } s > 1 \\ s, & \text{if } s \in [-1, 1] \\ -1, & \text{if } s < -1. \end{cases} \tag{5}$$

For simplicity, we denote y_{ji} as follows:

$$y_{ji}(t) := f_j(x_j(t)) - x_i(t). \tag{6}$$