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A collective neurodynamic optimization approach to bound-constrained nonconvex optimization[☆]

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This paper presents a novel collective neurodynamic optimization method for solving nonconvex optimization problems with bound constraints. First, it is proved that a one-layer projection neural network has a property that its equilibria are in one-to-one correspondence with the Karush–Kuhn–Tucker points of the constrained optimization problem. Next, a collective neurodynamic optimization approach is developed by utilizing a group of recurrent neural networks in framework of particle swarm optimization by emulating the paradigm of brainstorming. Each recurrent neural network carries out precise constrained local search according to its own neurodynamic equations. By iteratively improving the solution quality of each recurrent neural network using the information of locally best known solution and globally best known solution, the group can obtain the global optimal solution to a nonconvex optimization problem. The advantages of the proposed collective neurodynamic optimization approach over evolutionary approaches lie in its constraint handling ability and real-time computational efficiency. The effectiveness and characteristics of the proposed approach are illustrated by using many multimodal benchmark functions.

1. Introduction

Optimization problems are omnipresent in scientific and engineering applications including system modeling and control, signal processing, computer vision, pattern recognition, machine learning, and so on (e.g., see Boyd and Vandenberghe (2004), Hyvärinen and Oja (2000), Vapnik (2000)). As many real-world optimization problems become increasingly complex and fall into nonconvex optimization, computing global optimal solutions in real time using traditional numerical optimization techniques is computationally demanding. Developing better and efficient optimization methods is always desirable. A promising approach to real-time optimization is neurodynamic optimization where recurrent neural networks (RNNs) serve as parallel computational models for optimization problems solving (Xia & Wang, 1999). The essence of neurodynamic optimization lies in its inherent nature of parallel and distributed information processing and the availability of hardware implementation.

Since the pioneering work of Hopfield and Tank (1985) on using a neural network for solving the Traveling-Salesmen Problem, neurodynamic optimization has received a great deal of attention over the past three decades. Many researchers investigated alternative neurodynamic optimization models for solving various linear and nonlinear optimization problems. For example, Kennedy and Chua (1988) presented a recurrent neural network for nonlinear optimization by utilizing finite penalty parameter method to compute approximate optimal solutions. Zhang and Constantinides (1992) proposed a two-layer Lagrangian neural network to deal with the optimization problems with equality constraints. Wang (1994) proposed a deterministic annealing neural network for solving convex programming problems. Wang (1996) presented a recurrent neural network for solving the shortest path problem. Wang (1997) presented a primal–dual neural network for the zero–one integer linear programming. In recent years, many recurrent neural

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network models with simple model complexity and good convergence properties have been developed for linear programming (Liu & Wang, 2008a, 2011a), quadratic programming (Gao & Liao, 2010; Hu & Wang, 2008; Liu & Wang, 2006, 2008b; Xia, Feng, and Wang, 2004), variational inequalities (Hu & Wang, 2006, 2007), general convex nonlinear programming (Xia, Feng, & Wang, 2008; Xia, Leung, & Wang, 2002; Xia & Wang, 2004a, 2004b, 2005), pseudoconvex optimization (Guo, Liu, & Wang, 2011; Liu, Guo, & Wang, 2012), and nonsmooth optimization (Bian & Xue, 2009; Cheng et al., 2011; Forti, Nistri, & Quincampoix, 2004; Liu & Wang, 2011b, 2013). Most of the designed models are based on some classic optimality equations and conditions (Liao, Qi, & Qi, 2004). For example, Xia et al. (2002) applied a projection method for RNN design. Liu and Wang (2006) used duality properties. Liu and Wang (2008b) was based on the Karush–Kuhn–Tucker (KKT) conditions. Liu and Wang (2011a) applied saddle point theorems and obtained the finite-time convergence property. Cheng et al. (2011) applied the Lagrangian method for solving nonsmooth convex optimization problems. Hosseini, Wang, and Hosseini (2013), Liu and Wang (2011b), and Li, Yan, and Wang (2014) used nonsmooth penalty function methods for solving some generalized convex nonsmooth optimization problems.

While neurodynamic optimization approaches with individual RNNs have achieved great successes, they would reach their solvability limits at constrained optimization problems with unimodal objective functions exclusively and are impotent for global optimization with multimodal objective functions. When dealing with general nonconvex optimization problems, the dynamic behaviors of a recurrent neural network could change drastically and become unpredictable. In parallel to neurodynamic optimization research, population-based evolutionary computation approaches emerged as a branch of popular meta-heuristic methods for global and multi-objective optimization in recent years. Evolutionary optimization algorithms are stochastic, heuristic, discrete-time, and multiple-state in their nature. In particular, particle swarm optimization (PSO) is a global optimization method introduced by Kennedy and Eberhart that mimics swarm behaviors such as birds flocking and fish schooling (Eberhart & Kennedy, 1995; Kennedy & Eberhart, 1995). In recent years, a number of variant PSO algorithms have been proposed for the purpose of accelerating convergence speed and avoiding premature convergence. For example, Liang, Qin, Suganthan, and Baskar (2006) proposed a comprehensive PSO for discouraging premature convergence. Zhan, Zhang and Chung (2009) proposed an adaptive PSO for better search efficiency. Zhan, Zhang, Yi, and Shi (2011) proposed an orthogonal learning PSO for better solution quality and stronger robustness. Owing to its ease of implementation and high efficiency, PSO has been widely adopted and successfully applied for many optimization problems (Chen et al., 2010; Duan, Luo, Shi, & Ma, 2013; Eberhart & Shi, 2007).

Despite of their capabilities of global search, PSO algorithms are deficient in precise local search and constraint handling. In contrast, based on optimization and dynamic systems theories, neurodynamic optimization approaches are competent for constrained local search, but incapable of global search in the presence of nonconvexity. It would be a good idea to combine the two types of optimization methods for constrained global optimization. In this paper, a collective neurodynamic optimization approach in framework of PSO is proposed for optimization problems with bound constraints. Inspired by brainstorming, a number of RNNs are exploited in a cooperative way to tackle constrained nonconvex optimization problems. Each neural network carries out local search according to its own neurodynamics and converges to a candidate solution. The movements of the neural networks are guided by individual best known solution as well as the best known solution of the entire group. By iteratively improving the initial conditions and the converged solutions, the neural network group is expected to discover the global optimal solution. The proposed approach can be viewed as an emulation for the brainstorming process of human beings, which offers a new paradigm for real-time optimization.

The rest of this paper is organized as follows. In Section 2, a neural network model is reviewed. In Section 3, the optimality and convergence of the neural network are investigated. In Section 4, a collective neurodynamic optimization approach is developed. In Section 5, simulation results on benchmark problems are provided. Finally, Section 6 concludes this paper.

2. Problem formulation and model description

Consider an optimization problem with bound constraints

$$\begin{aligned} &\text{minimize} \quad f(x) \\ &\text{subject to} \quad x \in \Omega \end{aligned} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in \Re^n$ is the decision variable, $f : \Re^n \rightarrow \Re$ is an objective function, Ω is a nonempty and closed convex set in $\Re^n$. In this paper, Ω is assumed to be a box set.

Remark 1. Consider a general constrained optimization problem given by

$$\begin{aligned} &\text{minimize} \quad \tilde{f}(x) \\ &\text{subject to} \quad g_i(x) \le 0, i = 1, \ldots, m, \\ &x \in \Omega, \end{aligned} \tag{2}$$

where $g_i : \Re^n \rightarrow \Re, i = 1, \ldots, m$, are inequality constraints. One common approach to deal with the constrained optimization problem (2) is to introduce a penalty term into the objective function to penalize constraint violations (Fiacco & McCormick, 1990). The transformed objective function can be described as

$$f(x) = \tilde{f}(x) + \gamma\phi(g(x)), \tag{3}$$

where $\phi(g(x)) \ge 0$ is a real-valued function which imposes a penalty on constraint violation controlled by a penalty parameter γ. The penalty function method enables the solution to the optimization problem (2) to be obtained by solving the optimization problem (1) with a suitably defined exact penalty function. An exact penalty function has a property such that there exists a finite penalty parameter for which a solution to the penalized problem is a solution to the corresponding constrained problem. As a result, there is an incentive to develop a reliable and efficient approach to the optimization problem (1).

Xia et al. (2002) presented a projection based neural network for solving convex optimization problems subject to box constraints. The neural network model does not have any design parameter and hence it is more convenient for implementation. The neural network model is briefly reviewed here.

The one-layer projection neural network for the optimization problem (1) is described by the following dynamical equation:

$$\dot{x}(t) = -x(t) + P_\Omega(x(t) - \nabla f(x(t))), \tag{4}$$

where $x \in \Re^n$ is the state vector of the neural network, which corresponds to the decision vector in (1), ∇f is the gradient of f, and P_Ω is a projection operator defined as

$$P_\Omega(u) = \arg\min_{v \in \Omega} ||u - v||. \tag{5}$$

Computing the projection of a point onto a convex set is generally nontrivial. However, if Ω is a box set or a sphere set, the projection is well defined. When Ω is a box set, i.e., $\Omega = \{u \in \Re^n : l_i \le u_i \le h_i, i = 1, \ldots, n\}$, P_Ω is defined as

$$P_\Omega(u_i) = \begin{cases} l_i, & u_i < l_i; \\ u_i, & l_i \le u_i \le h_i; \\ h_i, & u_i > h_i. \end{cases} \tag{6}$$