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Sudoku associative memory

Jiann-Ming Wu [a,*], Pei-Hsun Hsu [b], Cheng-Yuan Liou [b]

[a] Department of Applied Mathematics, National Dong Hwa University, Shoufeng, Hualien 974, Taiwan
[b] Department of Computer Science and Information Engineering, National Taiwan University, Taiwan

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ABSTRACT

This work presents bipolar neural systems for check-rule embedded pattern restoration, fault-tolerant information encoding and Sudoku memory construction and association. The primitive bipolar neural unit is generalized to have internal fields and activations, which are respectively characterized by exponential growth and logistic differential dynamics, in response to inhibitory and excitatory stimuli. Coupling extended bipolar units induces multi-state artificial Potts neurons which are interconnected with inhibitory synapses for Latin square encoding, K-alphabet Latin square encoding and Sudoku encoding. The proposed neural dynamics can generally restore Sudoku patterns from partial sparse clues. Neural relaxation is based on mean field annealing that well guarantees reliable convergence to ground states. Sudoku associative memory combines inhibitory interconnections of Sudoku encoding with Hebb's excitatory synapses of encoding conjunctive relations among active units over memorized patterns. Sudoku associative memory is empirically shown reliable and effective for restoring memorized patterns subject to typical sparse clues, fewer partial clues, dense clues and perturbed or damaged clues. On the basis, compound Sudoku patterns are further extended to emulate complex topological information encoding.

1. Introduction

Studies on computational human intelligence have related memory association (Hopfield, 1982, 2008) to information processes of retrieving the stored full pattern from neural relaxation through synapses subject to partial clues. The research line mainly addresses on organizing essential neural circuits of promoting associative memory systems and intelligent agents. Associative memory (Amari, 2013; Gardner, 1988; Hopfield, 1982; Müller & Reinhardt, 1990; Palm, 2013) traditionally encodes unrestricted binary patterns for maximal storage utilization. Since requiring additional check bits for fault tolerance information storage or transmission, associative memory of encoding unrestricted binary patterns has been recently shifted to encoding check-rule embedded patterns. In the previous work (Hopfield, 2008), circumspectly designed bipolar neural circuits have been presented for general Sudoku resolution and attained not-always-correct empirical solutions. This pioneers the research direction of constructing bipolar neural circuits for reliable and effective error-detection, automatic error-correction and check-rule embedded pattern association.

This work presents two primitive differential dynamics for characterizing unbounded exponential internal fields and bounded logistic responses of extended bipolar processing units and coupling mechanism for organizing MIMO (multiple inputs and multiple outputs) (Salgado & Conley, 2004) units and multi-state artificial Potts neurons. Each extended bipolar unit possesses its unbounded internal field and logistic activation. The strength of an unbounded internal field inhibits or negatively contributes logistic activations of other coupled bipolar units. The emulated competitive mechanism conducts exclusive multi-state activations following the winner-take-all principle. The exclusive multi-state activation of coupled bipolar units sketches the simplest type of check-rule embedded binary patterns. The motivation of current research is to investigate feasibility of combining check-rule embedded pattern encoding with Hebb's rules for organizing Sudoku associative memory and explore effectiveness and reliability of the proposed Sudoku associative memory.

Artificial Potts neurons are equipped with inhibitory and excitatory interconnections for constructing Sudoku associative memory. The first step persuades reliable check-rule embedded pattern restoration for binary Latin square encoding. Among distinct Potts neurons, bipolar units that represent an identical state are fully connected with inhibitory synapses. The feasible configuration of K interconnected artificial Potts neurons is a binary Latin square

* Corresponding author. Tel.: +886 3 8633531; fax: +886 3 8633510.
E-mail addresses: jmwu@livemail.tw, jmwu@mail.ndhu.edu.tw (J.-M. Wu).

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matrix which allows one and only one active bit in each row and column. Inhibitory interconnections reduce the size of the feasible space from K^K to $K!$. The feasible configuration is no more unrestricted. The embedded combinatorics naturally serve as implicit check rules for automatic error detection and correction.

A cubic network of bipolar neural circuits, consisting of $K \times K$ artificial Potts neurons on a planar square, is organized for K-alphabet Latin square encoding. Each row and column in a feasible configuration exactly contains K distinct alphabets. Among artificial Potts neurons in each row and column, bipolar units that indicate the same state are interconnected with inhibitory synapses for K-alphabet Latin square encoding. $K \times K$ artificial Potts neurons can be equally partitioned to K non-overlapping blocks, each being a $R \times R$ planar square where R denotes the integer square root of K. Further posing block constraints that insist on K distinct alphabets in each block induces Sudoku encoding. A Sudoku puzzle refers to a K-alphabet Latin square that satisfies block constraints. Like Potts encoding (Wu, 2002), binary Latin square encoding (Kautz & Singleton, 1964), K-alphabet Latin square encoding (Kiefer & Wynn, 1981) and Sudoku encoding have self-contained check rules for error detection, pattern restoration and fault tolerance information processes.

The proposed bipolar neural units for Sudoku encoding relax through inhibitory interconnections to realize check-rule embedded pattern restoration subject to partial clues. Neural relaxation of restoring Sudoku pattern is ensured by mean field annealing that is oriented from statistical mechanism. It is remarkable that variant check-rules induce different types of feasible configurations. The complexity of embedded combinatorics definitely reflects robustness of pattern restoration and association against storage utilization. This work proposes novel neural systems for general Sudoku restoration and Sudoku memory association, where bipolar neural units are simultaneously equipped with inhibitory interconnections of Sudoku encoding and excitatory interconnections following Hebb's rules of memorizing designate Sudoku patterns. Sudoku associative memory can retrieve a memorized Sudoku pattern subject to fewer partial clues, dense clues even perturbed or damaged clues, significantly renovating traditional associative memory of unrestricted encoding. For theoretical memory association, Hebb's rules (Hebb, 1949) relate excitatory synaptic strength to simultaneous firing of two connected bipolar units. The synaptic interconnection is positively strengthened if one unit repeatedly assists in firing another. It turns to accumulate simultaneous activations of pair-wise units over memorized patterns to figure out excitatory synapses. A cubic network of bipolar units proposed for Sudoku associative memory contain excitatory synapses derived by averaging the outer product of each memorized pattern as well as inhibitory synapses of encoding constraints for general Sudoku restoration. Numerical simulations show Sudoku associative memory reducing necessary sparse clues of restoring the most difficult Sudoku puzzles more than fifty percents in average. Restoring memorized Sudoku patterns is no more restricted by the fewest number of clues claimed for unique Sudoku resolution (Delahaye, 2006). Besides, Sudoku associative memory can automatically correct perturbed partial clues for memory association, unfolding more than 30% fault tolerance to given typical clues.

This work applies Sudoku associative memory for regenerating compound Sudoku patterns. Each compound pattern consists of multiple Sudoku patterns that overlap common subgrids for planar expansion. For regeneration of full compound patterns, Sudoku memory association is employed to restore an unsolved Sudoku pattern subject to dense clues or a portion of common subgrids provided by one completely restored pattern. The overlapping portion of two Sudoku patterns is composed of two or three consecutive outer blocks in this work. Numerical simulations show total restoration of compound Sudoku patterns in V-shape or starfish-shape subject to few partial clues for regeneration. Sudoku associative memory promotes Sudoku restoration and compound pattern regeneration subject to fewer partial perturbed clues.

This paper devises cubic bipolar neural circuits for Sudoku resolution and check-rule embedded pattern association. Section 2 presents two sets of primitive differential dynamics for characterizing unbounded exponential internal fields and bounded logistic activations, coupling mechanism of extended bipolar units for Potts encoding, and inhibitory synapses of artificial Potts neurons for binary Latin square encoding and K-alphabet Latin square encoding. Section 3 recruits block constraints for Sudoku encoding. Section 4 presents Sudoku associative memory, Section 5 gives numerical simulations of Sudoku restoration and memory association and Section 6 explores memory capacity. Design and regeneration of compound Sudoku patterns are presented in Section 7 and conclusions are given in the last section.

2. Check-rule embedded neural encoding

2.1. Exponential growth and logistic dynamics

Hopfield neural networks (Hopfield, 1982) and feedforward multilayer perceptrons (Rosenblatt, 1962) are composed of bipolar neural processing units. A bipolar unit generates a two-alternative activation or continuous firing rate s in response to an external field h that integrates excitatory and inhibitory stimuli. For discrete case, it activates a maximal firing rate, if h exceeds a threshold, and a minimal firing rate otherwise. The two-alternative response to h coincides with the input–output relation of Widrow's adalines (Widrow, 1990).

In continuous case, the change of the membrane voltage and the firing rate of a bipolar unit is characterized by the primitive differential equations. An authentic model relates neural excitation u to an internal field or membrane potential (Dayan & Abbott, 2001) in change with respect to h proportional to its current value,

$$\frac{du(h)}{dh} = \beta u(h). \tag{1}$$

This induces an unbounded internal field, which should not be directly related to the bounded response to high external fields.

A reversion recruits a decay factor for compensating unbounded exponential growth. The derivative of firing rate s with respect to h is revised to be proportional to the product of $s(h)$ and $1 - s(h)$, where the decay factor measures the difference between the current firing rate and its upper bound, for preventing from unbounded exponential growth. The dynamic of continuous activations of bipolar processing units is expressed by the following equation,

$$\frac{ds(h)}{dh} = \beta s(h)\left[1 - s(h)\right]. \tag{2}$$

The derivative reaches the maximum when s approaches $\frac{1}{2}$ and the minimum when the firing rate approaches 0 or 1. The positive β modulates the smoothness of neural activations. When β equals 1, the derivative (2) is identical to the logistic differential equation (Petropoulou, 2010).

In general, the activation in response to the null external field, denoted by $s(0)$, states the initial condition of Eq. (2), such as

$$s(0) = \frac{1}{1+c}, \tag{3}$$

where c is positive and plays a role of shifting neural activations in response to $h = 0$. The mapping from h to s with different c is shown in Fig. 1. Larger c causes lower response to the null external field, equivalently shifting the activation curve rightward and inhibiting the firing. The sigmoid function of perceptrons or bipolar processing units of Hopfield neural networks can be proved as a special case with $c = 1$.