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Neural network for solving Nash equilibrium problem in application of multiuser power control

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ABSTRACT

In this paper, based on an equivalent mixed linear complementarity problem, we propose a neural network to solve multiuser power control optimization problems (MPCOP), which is modeled as the noncooperative Nash game in modern digital subscriber line (DSL). If the channel crosstalk coefficients matrix is positive semidefinite, it is shown that the proposed neural network is stable in the sense of Lyapunov and global convergence to a Nash equilibrium, and the Nash equilibrium is unique if the channel crosstalk coefficients matrix is positive definite. Finally, simulation results on two numerical examples show the effectiveness and performance of the proposed neural network.

1. Introduction

Due to low infrastructure cost and high speed data communication, digital subscriber line (DSL) technology has become a widely-used method for broadband internet access. In the DSL, power control of the design of interference-limited multiuser communication systems is a central issue, and it also has attracted significant attention from both academia and industry. Power control refers to forcing each user to transmit enough power so that it can achieve the required quality without causing unnecessary interference to other users in these systems. Therefore, the system design involves a performance tradeoff among the different users.

In the DSL, multiuser power control is an optimization problem, and a typical measure of system throughput is the sum of all users' rates (Cendrillon, Moonen, Verliden, Bostoen, & Yu, 2004; Cherubini, Eleftheriou, & Olcer, 2000; Song, Chung, Ginis, & Cioffi, 2002). Unfortunately, in this framework, the optimization problem of maximizing the sum rates is nonconvex with many local solutions (Song et al., 2002). In Cherubini et al. (2000), Cherubini et al. proposed a simulated annealing method to obtain a global optimal power allocation solution, which suffers from slow convergence and lack a rigorous analysis. Another method which has been very successful in solving multiuser power control is the game theoretic approach. Yu, Ginis, and Cioffi (2002) considered the multiuser power control problem in a frequency-selective interference channel, which is modeled as a noncooperative game, and its key observation is that each DSL user's data rate is a concave function of its own power spectra vector when the interfering users' power vectors are fixed. Luo and Pang (2006) presented a convergence analysis of iterative water-filling algorithm in more realistic channel settings and for arbitrary number of users. In Yamashita and Luo (2008), multiuser power control problem was formulated to find a Nash equilibrium of the DSL game as a nonlinear complementarity problem. In Cendrillon, Yu, Moonen, Verlinden, and Bostoen (2010), Cendrillon et al. discussed a centralized algorithm for optimal spectrum balancing. In Pang, Scutari, Facchinei, and Wang (2008), Pang et al. introduced the minimization of transmit power in Gaussian parallel interference channels subject to a rate constraint for each user. Therefore, many researchers have made deep research into the algorithm of multiuser power control optimization problem in the DSL.

In the application of multiuser power control optimization problems (MPCOP), real-time solutions are often needed. However, classical optimization methods are not competent for problems with high dimensionality or stringent computation time requirement. In the past two decades, the essence of neural network optimization lies in its inherent nature of parallel and distributed information processing and the availability of hardware implementation (Hopfield & Tank, 1985; Tank & Hopfield, 1986).

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Since the seminal work of Hopfield and Tank, there has been increasing interesting in investigating the theory, methodology and applications of recurrent neural networks for optimization. Based on penalty functions, Lagrange functions and primal and dual functions, many neural networks for solving various optimization problems (Bian & Chen, 2012; Cheng, Hou, & Tan, 2008; Forti, Nistri, & Quincampoix, 2006; Gao, Liao, & Qi, 2005; He, Li, Huang, & Li, 2014a, 2014b; Hosseini, Wang, & Mohamma, 2013; Hu & Wang, 2006, 2007; Hu & Zhang, 2009; Liu, Cao, & Chen, 2010; Liu, Guo, & Wang, 2012; Liu & Wang, 2011, 2013; Wang, 1993; Xia, 1996; Xia, Leung, & Wang, 2002; Xia & Wang, 2004) were developed, and these neural networks improved performance in terms of global convergence and parallel computational implementability.

In this paper, motivated by the effectiveness and efficiency of neural network optimization method, we have attempted to solve MPCOP using neural network approach. Compared with many iterative algorithms, our contribution is to design neural network for solving multiuser power control problem. Following the method (Luo & Pang, 2006), the Karush–Kuhn–Tacker optimality conditions of the noncooperative Nash game which results from MPCOP are reformulated as equivalent mixed linear complementarity problem (LCP). Based on LCP, a projection neural network is proposed for solving this Nash game. Using Lyapunov function theory, it is shown that the proposed neural network is Lyapunov stable and globally convergent to Nash equilibria sets, and under arbitrary symmetric interference environment and certain asymmetric channel condition, it is proven that the proposed neural network is globally convergent to unique Nash equilibrium. Finally, simulation results on numerical examples show the effectiveness and performance of the neural network for solving MPCOP.

The remainder of this paper is organized as follows. In the next section, the noncooperative Nash game about MPCOP is described, and a projection neural network is proposed to solve the noncooperative Nash game. The convergence of the proposed neural network is proved in Section 3. In Section 4, simulation results on two numerical examples are given. Finally, Section 5 concludes this paper.

Notation: Given column vectors $x = (x_1, x_2, \ldots, x_n)^T$, $(x)^+ = \left((x_1)^+, (x_2)^+, \ldots, (x_n)^+\right)^T$, $(x_i)^+ = \max(0, x_i)$, $1_n = \underbrace{[1, 1, \ldots, 1]}_{n}^{T}$, $x > 0$ means all the $x_i > 0$. $\|x\|$ denotes l_2 norm. $\forall a,\ b \in R^1, a \perp b$ denotes $a \cdot b = 0$. For $K \subset R^n$, the projection operator $H_K(x)$ is defined by: $H_K(x) = \arg\ \min_{y \in K} \|x - y\|$.

2. Problem formulation and model description

In this section, we introduce the game theoretic formulation of MPCOP in the DSL model. Then we will construct a recurrent neural network for solving MPCOP. Consider a tuple $(m, \mathcal{F}, \Omega)$ where m is the number of users in digital subscriber lines, n denotes the total number of frequency tones available to the DSL users. $\mathcal{F} = \left\{(f_i)_{i=1}^m\right\}$ is the set of user-specific objective functions, each user controls the variable $p^i = \left(p_1^i, p_2^i, \ldots, p_n^i\right)^T \in R^n$, which denotes the power spectra vector of user i with p_k^i signifying the power allocated to frequency tone k, and we denote by p the overall vector of all variables $p = \left((p^1)^T, (p^2)^T, \ldots, (p^m)^T\right)^T \in R^{mn}$, and the user-specific strategy sets Ω are denoted by $\Omega = \{\Omega_1, \Omega_2, \ldots, \Omega_m\}$, $\Omega_i \subseteq R^n$, and Ω_i is described by

$$\Omega_i = \left\{p^i \in R^n \mid 0 \le p_k^i \le CAP_k^i,\ \forall k = 1, \ldots, n, \sum_{k=1}^n p_k^i \le P_{\max}^i\right\},$$

where CAP_k^i and $P_{\max}^i$ are some positive constants. Taking p_k^j for $j \neq i$ as fixed, user i solve the following concave maximization problem

$$\max_{p^i} f_i\left(p^i; p^{-i}\right) \equiv \sum_{k=1}^n \log\left(1 + \frac{p_k^i}{\sigma_k^i + \sum_{j \neq i} \alpha_k^{ij} p_k^j}\right) \quad \text{s.t. } p^i \in \Omega_i, \quad (1)$$

where $p = \left(p^i; p^{-i}\right)$, p^i is the strategy of player i and p^{-i} are the strategies of all the users except i, σ_k^i are positive scalars and α_k^{ij} are nonnegative scalars for all $i \neq j$, and all k representing noise power spectra and channel crosstalk coefficients, respectively. Then a Nash equilibrium is a feasible $p^* = \left(\left(p^{1,*}\right)^T, \left(p^{2,*}\right)^T, \ldots, (p^{m,*})^T\right)^T$ such that $f_i\left(p^{i,*}; p^{-i,*}\right) \ge f_i\left(y^i; p^{-i,*}\right)$, $\forall y^i \in \Omega_i$, $\forall i = 1, 2, \ldots, m$. In the following, we state some assumptions about the cost functions and strategy sets.

Assumption 1. In the feasible set Ω_i, $P_{\max}^i < \sum_{k=1}^n CAP_k^i$.

Assumption 2. $\alpha_k^{ii} = 1$ for all k and i.

For problem (1), define a Lagrange function

$$L_i\left(p^i, u^i, \gamma^i\right) = -f_i + u^i\left(\sum_{k=1}^n p_k^i - P_{\max}^i\right) + \sum_{k=1}^n \gamma_k^i\left(p_k^i - CAP_k^i\right), \quad (2)$$

where $u^i \in R^1$, $\gamma_k^i \in R^1$ are the multiplier of the inequality $\sum_{k=1}^n p_k^i \le P_{\max}^i$, $p_k^i \le CAP_k^i$, respectively. According to the well-known saddle point theorem (Body & Vandenberghe, 2003), we can get the KKT conditions for user i of problem (1) as follows:

$$\begin{aligned} & 0 \le p_k^i \perp -\frac{1}{\sigma_k^i + \sum_{i=1}^m \alpha_k^{ij} p_k^j} + u_i + \gamma_k^i \ge 0, \quad \forall k = 1, 2, \ldots, n \\ & 0 \le u^i \perp P_{\max}^i - \sum_{k=1}^n p_k^i \ge 0 \\ & 0 \le \gamma_k^i \perp CAP_k^i - p_k^i \ge 0,\ \forall k = 1, 2, \ldots, n \end{aligned} \quad (3)$$

p^* is a solution of problem (1) if and only if there exist $u^* = \left(u^{1,*}, u^{2,*}, \ldots, u^{m,*}\right)^T \in R^m$ and $\gamma^* = \left(\left(\gamma^{1,*}\right)^T, \left(\gamma^{2,*}\right)^T, \ldots, (\gamma^{m,*})^T\right)^T \in R^{mn}$ such that (p^*, u^*, γ^*) satisfies system (3). From Luo and Pang (2006), we have the following results.

Lemma 1. *Under Assumptions 1–2, system (3) is equivalent to the following system*

$$\begin{aligned} & 0 \le p_k^i \perp \sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j + \upsilon_i + \varphi_k^i \ge 0, \quad \forall k = 1, 2, \ldots, n \\ & P_{\max}^i - \sum_{k=1}^n p_k^i = 0,\ \upsilon_i \text{ is free} \\ & 0 \le \varphi_k^i \perp CAP_k^i - p_k^i \ge 0, \quad \forall k = 1, 2, \ldots, n \end{aligned} \quad (4)$$

where $\upsilon_i = -\frac{1}{u_i}$, $\varphi_k^i = \frac{\gamma_k^i\left(\sigma_k^i + \sum_{j=1}^m \alpha_k^{ij} p_k^j\right)}{u_i}$, *and* p^* *is a solution of problem (1) if and only if there exist* $\upsilon^* = \left(\upsilon^{1,*}, \upsilon^{2,*}, \ldots, \upsilon^{m,*}\right)^T \in R^m$ *and* $\varphi^* = \left(\left(\varphi^{1,*}\right)^T, \left(\varphi^{2,*}\right)^T, \ldots, (\varphi^{m,*})^T\right)^T \in R^{mn}$ *such that* $(p^*, \upsilon^*, \varphi^*)$ *satisfies system (4).*