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A new switching design to finite-time stabilization of nonlinear systems with applications to neural networks

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ABSTRACT

This paper is concerned with the optimal finite-time stabilization problem for nonlinear systems. For the given stabilization strength, a new switching protocol is designed to stabilize the system with a fast speed. The obtained protocol covers both continuous control and discontinuous one under the framework of Filippov solutions. Some criteria are discussed in detail on how to choose an optimal protocol such that the finite stabilization time can be shortened. Finally, the main theory results are applied to the general neural networks by one numerical example to illustrate the effectiveness of the proposed design method.

1. Introduction

The problems of stability analysis and stabilization of nonlinear systems have received much attention in the past decades (Guo, Wang, & Yan, 2013; Hu, Gao, & Zheng, 2008; Liu, Lu, & Chen, 2013; Liu, Wang, & Liu, 2009; Shu, Lam, & Xiong, 2010; Wang, Lu, & Chen, 2010; Xu & Lam, 2006; Zeng & Wang, 2006), which play important roles in both control theory and system identification. Recently, based on different convergence time, the issue of stabilization has been classified into two types: one is the infinite time stabilization, such as exponential or asymptotic stabilization (Huang, Huang, Chen, & Qian, 2013; Lu, Ho, & Wang, 2009; Lu, Ho, & Wu, 2009); the other is *finite-time stabilization* (FTS), see Huang, Lin, and Yang (2005), Nersesov and Haddad (2008), Orlov, Aoustin, and Chevallereau (2011), Shen and Xia (2008), Zhang, Feng, and Sun (2012) and the references therein. In contrast to the commonly concept of asymptotic stability, finite-time stability requires essentially that a control system be Lyapunov stable and its trajectories tend to equilibrium in finite time. The FTS has drawn an increasing attention because the finite-time convergence demonstrates some nice features such as faster convergence and robustness to uncertainties (Hong & Jiang, 2006; Qian & Li, 2005).

In recent years, a Lyapunov theory has been presented for testing finite-time stability of continuous autonomous systems (Bhat & Bernstein, 1998, 2000; Haimo, 1986; Hong, Huang, & Xu, 2001; Khoo, Xie, & Man, 2009; Wang & Xiao, 2010; Xiao, Wang, Chen, & Gao, 2009), which provided a basic tool for analysis and synthesis of nonlinear control systems. On the other hand, the FTS could also be realized by non-smooth control, such as sliding mode control, and binary control (Chen, Lewis, & Xie, 2012; Cortés, 2006; Hui, Haddad, & Bhat, 2008; Niu & Ho, 2010; Niu, Ho, & Wang, 2008; Wu, Ho, & Li, 2011; Wu, Su, & Shi, 2012). From these references, we can summarize that most existing finite-time control protocols were designed as the form of $u(t) = -k\,\mathrm{sign}(y(t))|y(t)|^{\alpha}$, $0 \leq \alpha < 1$, where $y(t)$ denotes the system state and k the control strength. For different values of α, the control techniques are generally divided into two types: (i) *continuous* (when $0 < \alpha < 1$) (Khoo et al., 2009; Wang & Xiao, 2010; Xiao et al., 2009) and (ii) *discontinuous* (when $\alpha = 0$) (Chen et al., 2012; Cortés, 2006; Hui et al., 2008). It is natural to choose either one of these types as their approach to achieve the objectives of FTS. Nevertheless, such two types of control techniques are always discussed separately, there are few literature consider them concurrently (Levant, 1998). In addition, there is no existing result to integrate FTS with both continuous and discontinuous controls. Hence, we have the following question: *Does there exist any design method to realize FTS via both types of stabilization protocols with a faster convergence speed*? In this paper, in order to answer this question and to

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optimize the stabilization time, some criteria will be given here on choosing a new stabilization protocol to include the above two types under the same framework of Refs. Chen et al. (2012), Cortés (2006), Hui et al. (2008), Khoo et al. (2009), Wang and Xiao (2010) and Xiao et al. (2009).

It is well known that whether the stabilization protocol is continuous, the objective of finite-time stabilization can always be realized with sufficient large strength k in the stabilization protocol $u(t)$, and the larger k leads to faster convergence speed (Bhat & Bernstein, 1998, 2000; Chen et al., 2012; Cortés, 2006; Haimo, 1986; Hong et al., 2001; Hui et al., 2008; Khoo et al., 2009; Niu & Ho, 2010; Niu et al., 2008; Wang & Xiao, 2010; Wu et al., 2011, 2012; Xiao et al., 2009). In contrast, when the strength k is fixed, how to design an optimal finite-time stabilization protocol for the different value of α, has become a practical problem. Here, the "optimal" means choosing a proper α and obtaining the fast convergence speed under a given control strength. In fact, investigating such research problems would be difficult to decide whether a continuous or discontinuous finite-time stabilizer should be chosen. Does there exist a link between the continuous stabilizer and the discontinuous one? Hence, many open questions still remain unsolved. We shall address some of these non-trivial problems when considering the FTS for nonlinear systems:

- Q1 Since both the continuous and discontinuous control protocol can stabilize the nonlinear system in finite time, which one has the faster convergence speed under the same situation?
- Q2 If the designed control protocol is discontinuous, whether the classical solutions of this discontinuous system exist? Also, how to ensure this existence?
- Q3 Under the same control strength, how does the value of α affect convergence time? In order to minimize the bound of the stabilization time, how to design an optimal protocol?
- Q4 Can we design a common framework to accommodate both types of stabilizer? And then, can we find a new control protocol which could realize the FTS with a smaller bound of convergence time than that generated by each of them?

Motivated by the above questions, in this paper, we are interested in considering the stabilization protocol design and the FTS problems for nonlinear systems. The design objectives will be implemented step by step as follows:

- (a) Use a continuous control protocol $u(t)$ to realize FTS and obtain the upper bound of stabilization time T, which is a function of parameter α.
- (b) Calculate the minimum value of finite time $T(\alpha)$, which will be dependent on the size of initial state.
- (c) Design a new switching stabilization protocol, including continuous ($0 < \alpha < 1$) and discontinuous ($\alpha = 0$), such that the bound of the stabilization time is optimal.

The above questions are still open and have not been investigated by other researchers in existing literature. The contribution of this paper will shed some light on designing a stabilization protocol covering both the continuous and discontinuous two types. Moreover, convergence time has also become an important performance indicator when studying the stabilization of neural networks in recent years. And finite-time stabilization or synchronization for neural networks has been well studied with the continuous control protocol (Liu, Jiang, Cao, Wang, & Wang, 2013) and the discontinuous one (Shen & Cao, 2011), respectively. In the end of this paper, in order to demonstrate the effectiveness of the theory results, we will apply the new obtained control strategy to the FTS problem of neural networks by a numerical example.

The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I and 0 represent the identity matrix and a zero matrix, respectively; $\mathrm{diag}(\cdots)$ stands for a block-diagonal matrix; $\mathtt{sign}(\cdot)$ is the sign function. In symmetric block matrices or long matrix expressions, we use a star "$\star$" to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

The rest of this paper is organized as follows. Section 2 formulates the FTS problem and introduces some non-smooth dynamic system analysis tools. Section 3 presents the main results step by step, for FTS with continuous, discontinuous and switching stabilization protocol. Section 4 specializes a numerical example to apply the proposed control strategy to the FTS problem of a general neural network model, and the simulation results demonstrate the effectiveness of the proposed control methods. Section 5 gives some concluding remarks.

2. Model formulation and preliminaries

2.1. System description

In this paper, we consider the nonlinear systems described by the following differential equation

$$\dot{x}(t) = Ax(t) + Bf(x(t)) + J, \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ is the state vector; $f(x)$ is a nonlinear function; and J is the external disturbance vector.

In this paper, we want to control the nonlinear system (1) to the desired state x^*, which is an equilibrium point of (1). Without loss of generality, one can shift the equilibrium point x^* to the origin by using the translation $y(t) = x(t) - x^*$, which derives the following dynamic system:

$$\dot{y}(t) = Ay(t) + Bg(y(t)), \tag{2}$$

where $g(y(t)) = f(x(t)) - f(x^*)$.

Hence, in order to stabilize the system (1) to the equilibrium point x^*, equivalently, one can stabilize the nonlinear system (2) to the origin due to the transformation. In the remainder of this paper, a control protocol $u(t)$ will be designed for the stabilization of system (2). The controlled system can be described by the following differential equation

$$\dot{y}(t) = Ay(t) + Bg(y(t)) + u(t). \tag{3}$$

And the control protocol $u(t)$ is designed as follows:

$$u(t) = -k_1 y(t) - k_2 \mathtt{sign}(y(t))|y(t)|^{\alpha}, \tag{4}$$

where constants k_1, k_2 are control strength coefficients to be determined, the real number α satisfies $0 \leq \alpha < 1$.

Remark 1. If $0 < \alpha < 1$, $u(\cdot)$ is a continuous control protocol (CCP) in Khoo et al. (2009), Wang and Xiao (2010) and Xiao et al. (2009). If $\alpha = 0$, $u(\cdot)$ is indeed a discontinuous control protocol (DCP), which has been considered in Refs. Chen et al. (2012), Cortés (2006) and Hui et al. (2008). However, all these literature only discussed a single case for either $0 < \alpha < 1$ or $\alpha = 0$, respectively. In this paper, we will study the FTS problem under the framework of $0 \leq \alpha < 1$ where a new switching control protocol (SCP) will be designed to stabilize the system (1) or (2) in an optimal finite time.

Remark 2. At a later stage, we will see that the parameters k_1 and k_2 play different roles in ensuring FTS. In brief, the introduction of control strength k_1 is to guarantee the stability of the system as discussed in the other references, but the purpose of k_2 is to ensure this stability be realized in finite time. The detail analysis will be discussed in the end of Section 3.