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Neural networks letter

A note on finite-time and fixed-time stability☆

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ABSTRACT

In this letter, by discussing $\dot{t}(V) = \mu^{-1}(V)$, we provide a general approach to reveal the essence of finite-time stability and fixed-time convergence for the system $\dot{V}(t) = \mu(V(t))$. Thus, we derive some conditions for finite-time and fixed-time convergence. As applications, we propose schemes to achieve finite-time and fixed-time synchronization in complex networked systems.

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1. Introduction and preliminaries

In many practical situations, stability over a finite time interval is of interests rather than the classic Lyapunov asymptotic stability, since it is more physically realizable than concerning infinite time. There are two categories of concepts of stability over finite time interval. One is finite-time convergence that means that the system converges within a finite time interval for any initial values; the other is fixed-time convergence that means that the time intervals of convergence have a uniform upper-bounds for all initial values within the definitive domain. The previous works on this topic include Bhat and Bernstein (1998, 2000), Dorato (1961), Haimo (1986), Hong, Xu, and Huang (2002), and Roxin (1966).

The finite-time and fixed-time stability/convergence have been successfully applied in many fields. More related to the present letter, synchronization and consensus in networked systems have been attracting increasing interests (Lu & Chen, 2004, 2006). Many recent papers were concerned with proposing schemes to realize finite-time synchronization/consensus, see Cortés (2006); Jiang and Wang (2009, 2011); Liu, Lam, Yu, and Chen (2016); Meng, Jia, and Du (2016); Parsegov, Polyakov, and Shcherbakov (2012, 2013); Polyakov (2012); Polyakov, Efimov, and Perruquetti (2015); Wang and Hong (2010); Wang, Li, and Shi (2014); Zhao, Duan, and Wen (2015) and Zuo and Tie (2014) for references. The main techniques in these works depend on the candidate Lyapunov functions as well as its convergence. In this letter, we propose a simple, novel and general technique to re-visit the problem by regarding as an implicit inverse function of time and apply to the synchronization and consensus in networked system.

To exposit the idea, suppose a nonnegative scalar function $V(t)$ satisfies

$$\dot{V}(t) \le -\mu(V(t)). \tag{1}$$

Here, we make following assumptions:

(i) $V(t) \ge 0$ for all $t \ge 0$ and is differentiable with respect to t;
(ii) $\mu(V) > 0$ when $V > 0$ and $\mu(0) = 0$.

Proposition 1. *Suppose $V(t)$ satisfies*

$$\dot{V}(t) \le -\mu(V(t)), \tag{2}$$

and assumptions (i), (ii).

1. *if $t_1^*(V_0) = \int_0^{V_0} \frac{1}{\mu(V)} dV$ is finite, then $V(t) = 0$ for all $t \ge t^*(V_0)$;*
2. *if $t_2^* = \int_0^{\infty} \frac{1}{\mu(V)} dV$ is finite, then for any $V(0) = V_0 > 0$, $V(t) = 0$ for all $t \ge t_2^*$;*

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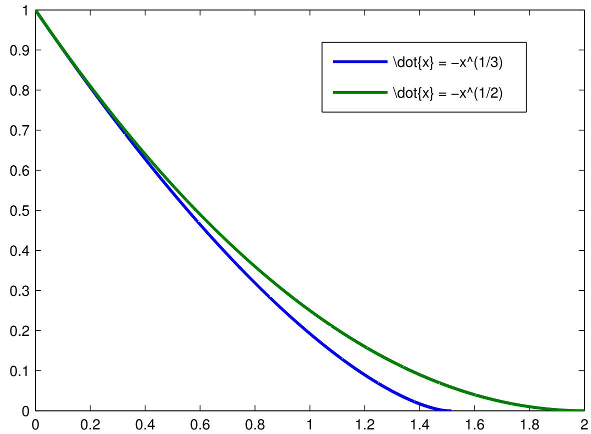

Fig. 1. Dynamical behavior of $\dot{t}(V) = \mu^{-1}(V)$.

Proof. Let $t(V)$ be the inverse function of $V(t)$, then $dt/dV \geq -\mu^{-1}(V)$, which implies

$$t \leq \int_{V(t)}^{V_0} \mu^{-1}(V)dV \leq t_1^*(V_0) \tag{3}$$

for all $V(t) \geq 0$, and $V(t_1^*(V_0)) = 0$. Therefore $V(t) = 0$ for all $t > t_1^*(V_0)$. Item (1) is proved.

Item (2) is a direct consequence of Item (1).

In addition, suppose

$$\dot{V}(t) = -\mu(V). \tag{4}$$

By similar arguments as above, it is easy to see that

$$t = \int_{V(t)}^{V_0} \mu^{-1}(V)dV.$$

Then,

$$\int_0^{V_0} \mu^{-1}(V)dV < \infty$$

is a necessary and sufficient condition that there is $t^* > 0$ such that $V(t) = 0$ for all $t \geq t^*$. And

$$\int_0^{\infty} \mu^{-1}(V)dV < \infty$$

is a necessary and sufficient condition that there is $t^* > 0$ such that $V(t) = 0$ for all $V(0)$ and $t \geq t^*$. □

It can be seen that the dynamics $\dot{V}(t) = -\mu(V)$ can also be written as $\dot{t}(V) = -\mu^{-1}(V)$. In this paper, instead of discussing original system, we discuss the system $\dot{t}(V) = -\mu^{-1}(V)$. In this way, the finite-time and fixed-time convergence can be derived easily (see Fig. 1).

Proposition 1 reveals that the finite-time convergence depends on the behavior of $\mu(V)$ in the neighborhood of $V = 0$. Instead, the fixed-time convergence depends on the behavior of $\mu(V)$ at both $V = 0$ and V approaching ∞ (see Fig. 2).

Assume $\mu(s) = \alpha s^p$. It is well known in case $0 < p < 1$,

$$\int_0^{V_0} V^{-p}dV = \frac{V^{1-p}(0)}{\alpha(1-p)} < \infty.$$

Instead, if $p \geq 1$, then

$$\int_0^{V_0} V^{-p}dV = \infty.$$

Therefore, in case $0 < p < 1$, the convergence is finite-time. Instead, if $p \geq 1$, the convergence is not finite-time.

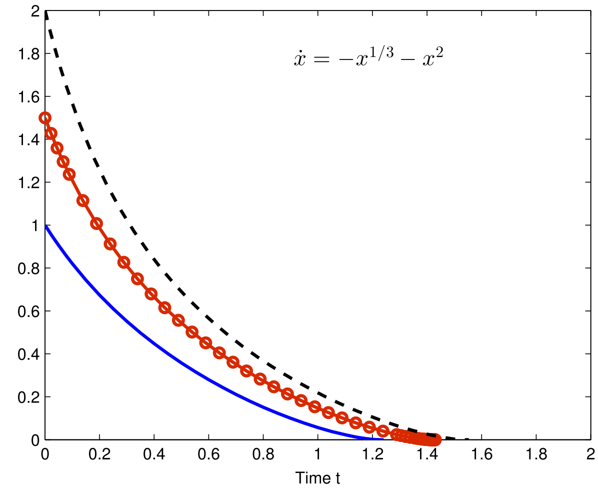

Fig. 2. Fixed-time convergence behaviors for different initial values.

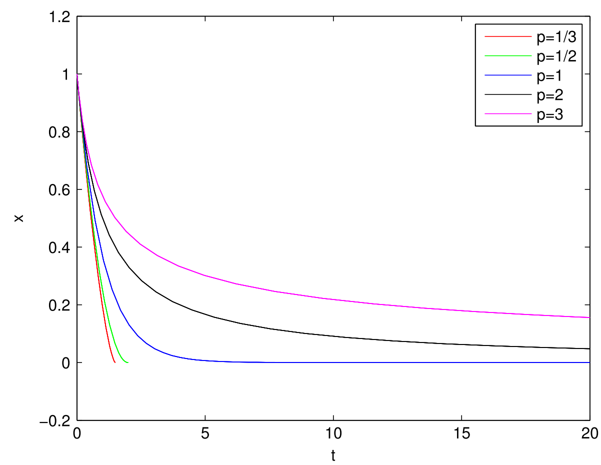

Fig. 3. Convergence behaviors for different indices "p".

In fact, when $p = 1$, the convergence is exponential.

If $p > 1$, then

$$t = \alpha^{-1} \int_{V(t)}^{V(0)} \frac{1}{V^p} dV = \frac{V^{1-p}(t) - V^{1-p}(0)}{\alpha(p-1)}$$

and

$$V(t) = [V^{1-p}(0) + \alpha(p-1)t]^{\frac{1}{1-p}}$$

which means that the convergence is with power rate $t^{-(p-1)}$ (see Chen & Wang, 2007).

The convergence for different p, finite-time convergence for $0 < p < 1$, exponential convergence for $p = 1$ and power-rate convergence for $p > 1$, is shown in Fig. 3.

With this approach, we have the following finite-time and fixed-time convergence result.

Theorem 1. *Suppose that the Dini derivative of a nonnegative function $V(t)$, denoted by D^+V, satisfies*

$$D^+V(t) \leq \begin{cases} -\mu_1(V(t)); & \text{if } 0 < V < a \\ -\mu_2(V(t)); & \text{if } V \geq a \end{cases} \tag{5}$$

for some constant $a > 0$, where functions $\mu_1(V) > 0$ and $\mu_2(V) > 0$ hold whenever $V > 0$; $\mu_1(0) = 0$. If

$$\int_0^a \frac{1}{\mu_1(V)} dV = \omega_1 < \infty, \qquad \int_a^{\infty} \frac{1}{\mu_2(V)} dV = \omega_2 < \infty,$$

then $V(t) = 0$ for all $t \geq \omega_1 + \omega_2$.

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