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Global exponential stability of impulsive complex-valued neural networks with both asynchronous time-varying and continuously distributed delays[☆]

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ABSTRACT

This paper investigates the stability problem for a class of impulsive complex-valued neural networks with both asynchronous time-varying and continuously distributed delays. By employing the idea of vector Lyapunov function, M-matrix theory and inequality technique, several sufficient conditions are obtained to ensure the global exponential stability of equilibrium point. When the impulsive effects are not considered, several sufficient conditions are also given to guarantee the existence, uniqueness and global exponential stability of equilibrium point. Two examples are given to illustrate the effectiveness and lower level of conservatism of the proposed criteria in comparison with some existing results.

1. Introduction

In the past three decades, neural networks had become a very attractive research field, because of their applications in many areas, for example, optoelectronics, imaging, remote sensing, quantum neural devices and systems, spatiotemporal analysis of physiological neural systems, artificial neural information processing, and other areas (Zeng & Zheng, 2013). In implementation of neural networks, because of finite switching speeds of the amplifiers, time delay is well-known to be unavoidable and it can cause oscillations or instability in dynamic systems (Arik, 2014). The stability as a very essential topic in neural networks with delays has attracted increasing interest, for example, see Balasubramaniam, Vembarasan, and Rakkiyappan (2012), Cao and Song (2006), Chen (2001), Kwon, Park, Lee, and Cha (2014), Ozcan and Arik (2014), Rakkiyappan, Sivasamy, Park, and Lee (2016) and references therein.

As applications of the neural networks spread more widely, developing neural network models which can deal with complex numbers is desired in various fields. Several models of complex-valued neural networks have been proposed and their abilities of information processing have been investigated (Hirose, 1992). Research has shown that complex-valued neural networks (CVNNs) make it possible to solve some problems which cannot be solved with their real-valued counterparts. For example, the XOR problem and the detection of symmetry problem cannot be solved with a single real-valued neuron, but they can be solved with a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potent computational power of complex-valued neurons (Jankowski, Lozowski, & Zurada, 1996).

For CVNNs, the main task is to find a suitable activation function in a variety of complex functions. In real-valued neural networks the activation functions are chosen to be smooth and bounded generally. However, the smooth and bounded functions cannot be chosen as activation functions of CVNNs according to Liouville's theorem since they will reduce to constants. Recently, there have been some researches on the stability of various CVNNs, for example, see Chen and Song (2013), Fang and Sun (2014), Gong, Liang, and Cao (2015a, 2015b), Hu and Wang (2012, 2015), Jankowski et al. (1996), Lee (2001), Liu and Chen (2016),

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Pan, Liu, and Xie (2015), Rakkiyappan, Velmurugan, and Cao (2015), Rakkiyappan, Velmurugan, and Li (2015), Rao and Murthy (2008), Song and Zhao (2016), Song, Zhao, and Liu (2015a, 2015b), Velmurugan, Rakkiyappan, and Cao (2015), Xu, Zhang, and Shi (2014), Zhang, Lin, and Chen (2014), Zhang and Yu (2016), Zhou and Song (2013), Zhou and Zurada (2009), and references therein. In Jankowski et al. (1996), authors proposed a CVNNs and supposed that its weight matrix was Hermitian with nonnegative diagonal entries in order to preserve the stability of the network. Then the assumption of weight matrix in Jankowski et al. (1996) was weakened by Lee (2001). In Fang and Sun (2014), Hu and Wang (2012), Zhang et al. (2014), Zhang and Yu (2016) and Zhou and Song (2013), the researchers investigated the asymptotical stability and exponential stability of CVNNs with constant delay. The CVNNs with time-varying delays were considered and some sufficient conditions for stability of a unique equilibrium were derived by Gong et al. (2015a, 2015b), Pan et al. (2015), Rakkiyappan, Velmurugan, and Cao (2015), Velmurugan et al. (2015). Based on delta differential operator, Chen and Song (2013) and Song and Zhao (2016) investigated the stability problem for a class of CVNNs with both leakage delay and time-varying delays on time scales. In Song et al. (2015a), a CVNNs model with probabilistic time-varying delays was established, and several sufficient conditions guaranteeing the global asymptotic and exponential stability of model were acquired. In Xu et al. (2014), the exponential stability of CVNNs with mixed delays was discussed. Hu and Wang (2015), Rao and Murthy (2008), Song et al. (2015b) and Zhou and Zurada (2009) also studied the stability of discrete-time CVNNs. Furthermore, impulsive effect on stability of CVNNs with time delays was considered by Rakkiyappan, Velmurugan, and Li (2015).

As pointed out in Liu and Chen (2016), the interconnections are generally asynchronous in practice, that is to say, the inevitable time delays between different nodes are generally different. For example, in order to model vehicular traffic flow (Bose & Ioannou, 2003; Helbing, 2001), the reaction delays of drivers should be considered, and for different drivers, the reaction delays are different depending on physical conditions, drivers' cognitive and physiological states. Moreover, in the load balancing problem (Chiasson et al., 2005), for a computing network consisting of n computers (also called nodes), except for the different communication delays, the task-transfer delays τ_{ij} also should be considered, which depend on the number of tasks to be transferred from node i to node j. Hence, based on the above discussions, it is necessary to study the dynamical behavior of neural networks with asynchronous time delays. In Liu and Chen (2016), the authors first proposed a complex-valued recurrent neural network model with asynchronous time delays, and presented several sufficient conditions for the uniqueness and global exponential stability of the equilibrium point by using three generalized norms.

It is well known that a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desired to model them by introducing continuously distributed delays over a certain duration of time such that the distant past has less influence compared with the recent behavior of the state (Song & Cao, 2006). Today, both time-varying delays and distributed delays have been widely accepted as important parameters associated with neural networks models, for example, see Jiang, Zeng, and Chen (2015), Song and Cao (2006), Wang, Liu, and Liu (2005) and references therein. On the other hand, the impulsive effects can be found in the similar way of time delay effect in the neural networks, in which many sudden and sharp changes occur instantaneously in the form of impulses in the particular neural networks (Rakkiyappan, Velmurugan, & Li, 2015). The impulsive perturbation of the neural networks can affect the dynamical behaviors, same as time delays effect. Recently, increasing attention has been focused on stability analysis of impulsive complex-valued neural networks and complex-valued systems with time delays, for example, see Rakkiyappan, Velmurugan, and Li (2015), Song et al. (2015b), Zeng, Li, Huang, and He (2015). However, to the best of the authors' knowledge, there are very few results on stability of impulsive CVNNs with both asynchronous time-varying and continuously distributed delays. This motivates our present research.

Motivated by the above discussions, the objective of this paper is to study the stability of a class of impulsive CVNNs with both asynchronous time-varying and continuously distributed delays. Applying the idea of vector Lyapunov function, M-matrix theory and inequality technique, we obtain several new sufficient conditions for checking the global asymptotic and exponential stability of CVNNs with asynchronous time-varying delays.

Notations: The notations are quite standard. Throughout this paper, i shows the imaginary unit, i.e., $i=\sqrt{-1}$. For complex number $z=x+iy$, the notation $|z|=\sqrt{x^2+y^2}$ stands for the module of z. E represents the unitary matrix with appropriate dimensions. $\mathbb{C}$, $\mathbb{C}^n$ and $\mathbb{C}^{n\times m}$ denote, respectively, the set of all complex numbers, the set of all n-dimensional complex-valued vectors and the set of all $n\times m$ complex-valued matrices. $\overline{A}$ and A^* show the conjugate and conjugate transpose of complex-valued matrix A, respectively. For a complex-valued vector $u=(u_1,u_2,\ldots,u_n)^T\in\mathbb{C}^n$, $|u|$ denotes the module vector given by $|u|=(|u_1|,|u_2|,\ldots,|u_n|)^T$, while the notation $\|u\|$ is the Euclidean norm of u. For a complex-valued matrix $A=(a_{ij})_{n\times n}\in\mathbb{C}^{n\times n}$, $|A|$ denotes the module matrix given by $|A|=(|a_{ij}|)_{n\times n}$, while $\|A\|$ denotes a matrix norm defined by $\|A\|=\sqrt{A^*A}$. $\rho(A)$ denotes the spectral radius of matrix A.

2. Model description and preliminaries

In this paper, we consider the following impulsive CVNNs with both asynchronous time-varying and continuously distributed delays

$$\begin{cases} \dot{z}_i(t)=-d_iz_i(t)+\sum_{j=1}^{n}a_{ij}f_j(z_j(t))+\sum_{j=1}^{n}b_{ij}f_j(z_j(t-\tau_{ij}(t))) \\ \qquad +\sum_{j=1}^{n}c_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(z_j(s))ds+I_i, \qquad t\neq t_k, \\ z_i(t)=p_{ik}(z_1(t^-),\ldots,z_n(t^-)) \\ \qquad +q_{ik}(z_1((t-\tau_{i1}(t))^-),\ldots,z_n((t-\tau_{in}(t))^-))+J_{ik}, \\ \qquad t=t_k \end{cases} \tag{1}$$

for $t\geq 0$, where $z_i(t)\in\mathbb{C}$ is the state of the ith neuron at time t; $f_j(z_j(t))\in\mathbb{C}$ and $f_j(z_j(t-\tau_{ij}(t)))\in\mathbb{C}$ are the activation functions without and with time delays; $\tau_{ij}(t)$ corresponds to the transmission delay along the axon of the jth unit from the ith unit and satisfies $0\leq\tau_{ij}(t)\leq\tau_{ij}$ (τ_{ij} is a constant); $D=\text{diag}\{d_1,d_2,\ldots,d_n\}\in\mathbb{R}^{n\times n}$ is the self-feedback connection weight matrix, where $d_i>0$; $A=(a_{ij})_{n\times n}\in\mathbb{C}^{n\times n}$, $B=(b_{ij})_{n\times n}\in\mathbb{C}^{n\times n}$ and $C=(c_{ij})_{n\times n}\in\mathbb{C}^{n\times n}$ are the connection weight matrices; $I=(I_1,I_2,\ldots,I_n)^T\in\mathbb{C}^n$ is the input vector; $K_{ij}:[0,+\infty)\to[0,+\infty)$ is the delay kernel function. The second part is discrete part of model (1), which describes that the evolution processes experience abrupt change of state at the moments of time t_k (called impulsive moments), where $z_j(t^-)$ and $z_j((t-\tau_{ij}(t))^-)$ denote the left limit of $z_j(t)$ and $z_j(t-\tau_{ij}(t))$, respectively; $p_{ik}(z_1(t^-),\ldots,z_n(t^-))$ represents impulsive perturbations of the ith unit at time t_k, and $q_{ik}(z_1((t-\tau_{i1}(t))^-),\ldots,z_n((t-\tau_{in}(t))^-))$ represents impulsive perturbations of the ith unit at time $t_k-\tau_{ij}(t_k)$; the fixed moments of time t_k satisfy $0<t_1<t_2<\cdots$, $\lim_{k\to+\infty}t_k=+\infty$.

The initial conditions of model (1) are in the form of $z_i(s)=\phi_i(s)$, $s\in(-\infty,0]$, where ϕ_i is bounded and continuous on $(-\infty,0]$.