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Stability analysis of switched stochastic neural networks with time-varying delays☆

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ABSTRACT

This paper is concerned with the global exponential stability of switched stochastic neural networks with time-varying delays. Firstly, the stability of switched stochastic delayed neural networks with stable subsystems is investigated by utilizing the mathematical induction method, the piecewise Lyapunov function and the average dwell time approach. Secondly, by utilizing the extended comparison principle from impulsive systems, the stability of stochastic switched delayed neural networks with both stable and unstable subsystems is analyzed and several easy to verify conditions are derived to ensure the exponential mean square stability of switched delayed neural networks with stochastic disturbances. The effectiveness of the proposed results is illustrated by two simulation examples.

1. Introduction

Neural networks have been extensively studied over the past few decades because of their applications in many areas, such as signal processing, associative memory, pattern recognition, combination optimization and so on (Haykin, 1999; Liu, Xiong, DasGupta, & Zhang, 2006; Rutkowski, 2004; Tang, Gao, & Kurths, 2013; Xia & Wang, 2004). In the implementation of artificial neural networks, time-delays are unavoidable due to the finite switching speed of amplifiers. It is well-known that time-delays may result in oscillation and instability (Liu, Lu, & Chen, 2011; Liu, Wang, Liang, & Liu, 2009; Niculescu & Gu, 2004). Moreover, noise disturbances can cause an instability and poor performance in neural networks. Actually, the synaptic transmission in real neural networks can be viewed as a noisy process introduced by random fluctuations from the release of neurotransmitters and other probabilistic causes (Cai, Huang, Guo, & Chen, 2012; Huang, Huang, & Chen, 2012; Liang, Wang, Liu, & Liu, 2008; Tang & Wong, 2013; Tang, Zou, Lu, & Kurths, 2012). Therefore, time-delays and noise disturbances can heavily affect the dynamical behaviors of neural networks, and thus it is necessary to investigate the effects of time-delays and noise disturbances on the stability of neural networks at the same time. In recent years, a variety of competing stability conditions have been established for stochastic delayed neural networks (Karimi & Gao, 2010; Li, Gao, & Shi, 2010; Shen & Wang, 2012; Tang, Gao, Kurths, & Fang, 2012; Tang, Wang, Gao, Swift, & Kurths, 2012; Wang, Liu, Li, & Liu, 2006; Zhang, Tang, Fang, & Wu, 2012; Zhang & Wang, 2008; Zhou, Tong, Gao, Ji, & Su, 2012; Zhou, Xu, Zhang, Zou, & Shen, 2012).

Among various types of neural networks, switched neural networks have been recognized to be a natural and exact way to model the phenomenon of information latching, and the abrupt phenomenon such as deterministic or random failures and the change of interconnections in subsystems (Lin & Antsaklis, 2009). Recently, the stability analysis of neural networks with Markovian switching or arbitrary switching have been investigated in Shen and Wang (2009), Tang, Gao, Zou, and Kurths (2013), Wu, Feng, and Zheng (2010); Wu, Shi, Su, and Chu (2011), Yang, Cao, and Lu (2012), Zhang and Gao (2010), Zhang and Shi (2009), Zhang and Yu (2009) and Zhu and Cao (2012). For neural networks with Markovian switching, the exponential stability for bidirectional associative memory and Cohen–Grossberg neural networks with Markovian switching was analyzed in Huang and Cao (2011) and Zhu and Cao (2012). In Shen and Wang (2009), the almost sure exponential stability of recurrent neural networks with Markovian switching was investigated by means of a generalized stochastic

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Halanay inequality. On the other hand, for neural networks with arbitrary switching, there has been an enormous growth of interest in using the average dwell time approach to deal with the stability problem of switched systems due to its effectiveness and simplicity (Li & Wang, 2013; Wu et al., 2010, 2011; Zhang & Gao, 2010; Zhang & Shi, 2009; Zhang, Tang, Miao, & Du, 2013; Zhang & Yu, 2009). For example, based on the average dwell time method, the exponential stability criteria were obtained for continuous-time switched neural networks with constant and time-varying delays in Wu et al. (2010). In Wu et al. (2011), the delay-dependent stability problem was examined for switched neural networks with time-varying delays, and the state decay estimate was explicitly given.

Most of the existing results are usually assumed that all the subsystems of switched systems are stable, which neglects the intrinsic features of practical switched systems. For example, subsystems might be unstable due to the failure of a component or abrupt disturbances. Therefore, it is more practical to consider switched systems with both stable and unstable subsystems (Li, Feng, & Huang, 2008; Liu & Marquez, 2007; Wei, Jie, & Ping, 2011; Wei & Zhang, 2006; Zhai, Hu, Yasuda, & Michel, 2001). The mean square stability for switched stochastic delay-free systems with both stable and unstable subsystems was analyzed by utilizing the Bellman–Gronwall inequality (Wei & Zhang, 2006), and the pth moment stability for switched stochastic delay-free systems was further obtained in Wei et al. (2011). In Li et al. (2008), the stability of impulsive switched delayed neural networks with both stable and unstable subsystems was investigated by using the dwell time approach. Two types of impulsive neural networks were considered: neural networks with only stable subsystems and neural networks with both stable and unstable subsystems, respectively. However, stochastic perturbations are neglected primarily due to their methods confined to analysis of deterministic systems. In addition to this limitation, for each subsystem, an upper and a lower bound are assumed to derive the stability of switched neural network. Moreover, almost all the research efforts on the stability of switched systems with both stable and unstable subsystems have assumed that the ratio of the total time running on all stable to unstable subsystems is less than a constant, this assumption implies that stable subsystems are active at first, then the unstable subsystems follow. However, in reality, it is impractical to ensure that stable subsystems can always be active at the beginning (Lin & Antsaklis, 2009).

The aforementioned discussion has motivated an investigation into the following questions: (1) Is it possible for us to carry out stability analysis of switched stochastic delayed neural networks with stable subsystems or switched stochastic delayed neural networks with both stable subsystems and unstable subsystems? (2) What kind of conditions can guarantee the exponential stability of switched stochastic delayed neural networks with both stable and unstable subsystems? (3) How do the systems' parameters, such as internal time-delays, the intensity of stochastic perturbations and the switching rules, affect the stability performance of switched stochastic delayed neural networks? (4) Can we establish stability criteria with relatively loose constraints of the subsystems' active time?

In this paper, the stability problem of global exponential stability for two types of switched stochastic neural networks are considered: systems with only stable subsystems and systems with both stable and unstable subsystems. By using the average dwell time approach, the comparison principle and the stochastic analysis techniques, the global exponential stability of switched stochastic delayed neural networks is investigated. The contributions of this paper are listed as follows: (1) the global exponential stability of switched stochastic neural networks with time-varying delays is investigated, which is more general and encompasses some recently developed models as special cases; (2) the stability problems are dealt with not only switched delayed neural networks with only stable subsystems, but also switched neural networks with both stable and unstable subsystems; (3) based on the comparison principle, a novel approach is proposed to deal with the stability analysis of switched stochastic delayed neural networks, and some easy to verified conditions are derived to remove some restrictive constraints on subsystems' active time.

The rest of this paper is organized as follows. In Section 2, the model of switched stochastic delayed neural networks is presented, together with some notations, definitions and lemmas. In Section 3, the global exponential stability criteria are derived for switched stochastic neural networks with time-varying delays. This section is divided into two subdivisions. The first subdivision focuses on the case of switched neural networks with stable subsystems, whereas the second subdivision concentrates on the case of switched neural networks with both stable and unstable subsystems. Finally, two simulation examples are given to illustrate the effectiveness of our results.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional and the set of all $n \times m$ matrices. Let $\mathbb{N} = 1, 2, \dots, \mathbb{R}^+ = [0, +\infty)$ and the superscript "T" denotes the transpose of a matrix or vector. For any matrix A, $\lambda_{\max}(A)$ denotes the largest eigenvalue of A and $\|A\| = \sqrt{\lambda_{\max}(A^TA)}$. Let $\omega(t) = [\omega_1(t), \omega_2(t), \dots, \omega_n(t)]^T$ be an n-dimension Brown motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a natural filtration (i.e., $\{\mathcal{F}_t\} = \sigma(\omega(s)) : 0 \le s \le t$). For $-\infty < a < b < \infty$, we say that a function from $[a, b]$ to $\mathbb{R}^n$ is piecewise continuous, if the function has at most a finite number of jumps discontinuous on $(a, b]$ and are continuous from the right for all points in $[a, b)$. Given $\tau > 0$, $PC([-\tau, 0]; \mathbb{R}^n)$ denotes the family of piecewise continuous functions from $[-\tau, 0]$ to $\mathbb{R}^n$ with norm $\|\varphi\| = \sup_{-\tau\le\theta\le 0} |\varphi(\theta)|$. Let $\mathcal{C}^{1,2}$ denote the family of all nonnegative functions $V(t, x, i)$ on $[t_0-\tau, \infty)\times\mathbb{R}^n\times\Gamma$ that are continuously once differentiable in t and twice in x. For $t \ge t_0$, let $PC^2_{\mathcal{F}_t}([-\tau, 0]; \mathbb{R}^n)$ be the family of $\mathcal{F}_t$-adapted and $PC([-\tau, 0]; \mathbb{R}^n)$-valued random variables φ such that $\mathbf{E}\|\varphi\|^2 < \infty$, where $\mathbf{E}$ denotes the expectation operator. For function $\psi : \mathbb{R} \to \mathbb{R}$, denote $\psi(t^-) = \lim_{s\to 0^-} \psi(t + s)$, and the Dini derivative of $\psi(t)$ is defined as $D^+\psi(t) = \lim\sup_{s\to 0^+}(\psi(t + s) - \psi(t))/s$.

2. Preliminaries

In this section, some preliminaries including model formulation, lemmas and definitions are presented.

Consider the following switched stochastic neural network with time-varying delays

$$\begin{aligned} dx(t) &= [-A_{\sigma(t)}x(t) + B_{\sigma(t)}f(x(t)) + C_{\sigma(t)}f(x(t-\tau(t)))]dt \\ &\quad + g(x(t), x(t-\tau(t)), \sigma(t))d\omega(t), \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector associated with the n neurons; $\sigma(t) : [0, \infty) \to \Gamma = \{1, 2, \dots, m\}$ is a piecewise constant function, continuous from the right, specifying the index of the active subsystem, i.e. $\sigma(t) = k_n \in \Gamma$ for $t \in [t_n, t_{n+1})$, where t_n is the nth switching time instant; A_i is a diagonal matrix with positive entries, B_i and C_i are the connection weight matrices, where $i \in \Gamma; f(x(t)) = (f_1(x(t)), f_2(x(t)), \dots, f_n(x(t)))^T$ denotes the neuron activation function; the noise perturbation $g : \mathbb{R}^n \times \mathbb{R}^n \times \Gamma \to \mathbb{R}^{n\times m}$ is a Borel function. $\tau(t)$ is a time varying delay satisfying $0 \le \tau(t) \le \tau$, and $\xi(t) = x(t-\tau(t)) \in PC^2_{\mathcal{F}_{t_0}}([-\tau, 0]; \mathbb{R}^d)$ is the initial value of (1). Let $x_t = x(t-\tau(t))$ for convenience. For the nonlinear function $f(\cdot)$ and the noise perturbation $g(\cdot)$, we make the following assumptions.