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Periodicity and global exponential stability of generalized Cohen–Grossberg neural networks with discontinuous activations and mixed delays[☆]

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ABSTRACT

In this paper, we investigate the periodic dynamical behaviors for a class of general Cohen–Grossberg neural networks with discontinuous right-hand sides, time-varying and distributed delays. By means of retarded differential inclusions theory and the fixed point theorem of multi-valued maps, the existence of periodic solutions for the neural networks is obtained. After that, we derive some sufficient conditions for the global exponential stability and convergence of the neural networks, in terms of nonsmooth analysis theory with generalized Lyapunov approach. Without assuming the boundedness (or the growth condition) and monotonicity of the discontinuous neuron activation functions, our results will also be valid. Moreover, our results extend previous works not only on discrete time-varying and distributed delayed neural networks with continuous or even Lipschitz continuous activations, but also on discrete time-varying and distributed delayed neural networks with discontinuous activations. We give some numerical examples to show the applicability and effectiveness of our main results.

1. Introduction

Neural networks with discontinuous (or non-Lipschitz, or non-smooth) neuron activations, have been found useful to address a number of interesting engineering tasks, such as dry friction, impacting machines, systems oscillating under the effect of an earthquake, power circuits, switching in electronic circuits, linear complementarity systems, and many others, and therefore much efforts have been devoted to analyzing the dynamical behavior of neural networks with discontinuous activations (Bartolini, Parodi, Utkin, & Zolezzi, 1999; Chong, Hui, & Zak, 1999; Cortés, 2008; Danca, 2002; Forti & Nistri, 2003; Forti, Nistri, & Papini, 2005; Jong et al., 2004; Utkin, 1977, 1978, 1992). In addition, the analysis of discontinuous neural network systems can reveal many specially interesting and important traits of the dynamics such as the phenomenon of convergence in finite time toward the equilibrium point or limit cycle. Thus, it is of practical importance to explore the dynamical behaviors of discontinuous neural network systems.

It is well known that, under the framework of the theory of Filippov differential inclusions, the paper (Forti & Nistri, 2003) is the first one to deal with the global stability of a neural network modeled by a differential equation with a discontinuous right-hand side. As Forti and Nistri (2003) pointed out, a brief review of some common neural network models reveals that neural network systems with discontinuous neuron activations are important and do frequently arise in the applications. In fact, consider the classical Hopfield neural networks with graded response neurons (Hopfield, 1984). Under the standard assumption of high-gain amplifiers, the neuron activations closely approach discontinuous and comparator functions. In addition, neuron activation with very-high gain is frequently encountered in the neural network applications for solving constrained optimization problems via a sliding mode approach (Chong et al., 1999; Forti, Nistri, & Quincampoix, 2004).

In the subsequent literature, many considerable efforts have been devoted to investigate the neural network system with discontinuous activation functions. In Forti, Grazzini, and Nistri (2006), Forti et al. (2005), Liu, Liu, and Xie (2012), Lu and Chen (2005), Nie and Cao (2012), Wang and Zhou (2012) and Xiao, Zeng, and Shen (2013), a series of results were obtained for the global stability of the unique equilibrium point of the delayed neural networks with discontinuous or non-Lipschitz activations. In Cai, Huang, Guo, and Chen (2012), Chen and Song

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(2010), Liu and Cao (2009), Wu and Li (2008), Wu and Shan (2009) and Xue and Yu (2007), by using the theory of fixed point in differential inclusion and Lyapunov-like approach, the authors analyzed the problems of global exponential stability of the periodic solutions for various neural network systems with discontinuous activations, respectively. In Lu and Chen (2008), Qin, Xue and Wang (2013), Wang and Huang (2012), the authors investigated the existence, uniqueness and global stability of almost periodic solutions for various neural network systems with discontinuous activations, respectively. However, all discussions in these papers are based on the assumption that the activation functions are monotonically nondecreasing. It is worth pointing out that, the detailed shape of the activation does not matter very much as long, since its growth is dominated by the passive decay that stabilizes the model.

To overcome this fault, the papers, such as Cai and Huang (2011), Guo and Huang (2009), Huang, Wang, and Zhou (2009), Li and Wu (2009), Liu and Cao (2010), Wang, Huang, and Guo (2009a,b) and Xiao et al. (2013), considered the following unilateral Lipschitz-like condition for the discontinuous activation functions g_i:

(H1) (unilateral Lipschitz-like condition): for each $i = 1, 2, \ldots, n$, there exists a constant L_i, such that for any two different numbers $u, v \in \mathbb{R}$, $\forall \gamma_i \in \overline{\text{co}}[g_i(u)]$, $\xi_i \in \overline{\text{co}}[g_i(v)]$,

$$\frac{\gamma_i - \xi_i}{u - v} \geqslant L_i$$

where $\overline{\text{co}}[g_i(u)] = [g_i^-(u), g_i^+(u)]$ and $g_i^-(u) = \lim_{s\to u^-} g(s)$, $g_i^+(u) = \lim_{s\to u^+} g(s)$. However, the unilateral Lipschitz-like condition for the discontinuous activation functions have often been severely criticized. One of the criticisms is that in such a model, the discontinuous activation functions should satisfy the restriction conditions $g_i^+(\rho_k^i) > g_i^-(\rho_k^i)$ (where $g_i(s)$ is discontinuous at point ρ_k^i). This assumption does not match with the actual. For example, see the Example 4.1 of Cai et al. (2012). In addition, as pointed out by Gonzalez (2000), to truly exploit the potential of neural networks, a nonlinear activation function must be used. Virtually all neural networks use nonlinear activation functions at some point within the network. When dealing with a dependent variable that is not bounded, we could choose an unbounded nonlinear activation function, such as $-x^3$, x^2, etc. Clearly, the activation functions $\frac{1}{1+e^{-\beta x}}$, $-x^3$, x^2 do not satisfy the unilateral Lipschitz-like condition.

Hence, the more practical and interesting model of neuron network is, in such model the discontinuous activation functions maybe not satisfy the nondecreasing or the unilateral Lipschitz-like condition. One of the contributions of this paper is: dropping the assumptions on monotonicity and unilateral Lipschitz-like condition of the discontinuous activation functions, we study the global stability of periodic solutions for neuron network systems.

Note that the properties of periodic solutions are of great interest, which have been successfully applied in many neural network systems, such as many biological and cognitive activities (for example heartbeat, respiration, mastication, locomotion, and memorization) require repetition. Periodic oscillations in recurrent neural networks have found many applications, such as associative memories (Nishikawa, Lai, & Hoppensteadt, 2004), pattern recognition (Chen, Wang, & Liu, 2000; Wang, 1995), machine learning (Ruiz, Owens, & Townley, 1998; Townley et al., 2000), and robot motion control (Jin & Zacksenhouse, 2003). In particularly, an equilibrium point can be regarded as a special case of periodic solution for neural networks with arbitrary period. Therefore, the analysis of periodic solutions for neural networks are more general and interesting.

In many practical applications of neural networks, for example, control, image processing, pattern recognition, signal processing and associative memory, time delays are often inevitable, since the finite switching speed of amplifiers and communication time. For instance, in the signals transmitted among the cells, one must introduce time delays to process moving images (see Civalleri, Gilli, & Pandolfi, 1993). As pointed by Jiang et al. that, time delayed neural networks can also capture the dynamic nature of speech to achieve superior phoneme recognition performance using standard error back-propagation (BP) (see Jiang, Gielen, Deng, & Zhu, 2002). For more knowledge about the practical design and application of time-delayed neural networks we refer to Chen and Song (2010), Foss, Longtin, and Milton (1996), Ghosh, Rho, Mclntosh, Kötter, and Jitsa (2008), Izhikevich (2006) and Marcus and Westervelt (1989) etc. According to Chen and Song (2010), time delays can affect the stability of the neural network systems and may lead to some complex dynamic behaviors such as oscillation, chaos and instability. Moreover, the issue of stability analysis of the discontinuous dynamical systems is also a significant research topic in neural theory. Thus, it is of great importance to explore the dynamical behaviors of neural networks with time delays, such as the existence, uniqueness, stability and global exponential stability of periodic solutions, etc.

Moreover, as pointed by Forti et al., it is interesting and important to investigate discontinuous neural networks with more general delays, such as time-varying or distributed ones. For example, in electronic implementation of analog neural networks, the delays between neurons are usually time-varying and sometimes vary violently with time due to the finite switching speed of amplifiers and faults in the electrical circuit (see Hou & Qian, 1998 and Huang, Ho, & Lam, 2005). Thus, we consider the more general type of delays, such as time-varying and distributed ones, which are general more complex and therefore more difficult to deal with. From the theoretical point of view, when the time-varying delays and distributed delays are introduced into the discontinuous neuron activations, the theory of differential inclusions with memory (i.e. functional differential inclusions) is a standard and effective tool to deal with the problems of dynamical behaviors for neural networks modeled by time-varying and distributed differential equations with discontinuous right-hand sides. According to Aubin and Cellina, functional differential inclusions express that the velocity depends not only on the state of the system at every instant, but depends upon the history of the trajectory until this instant (Aubin & Cellina, 1984). Generally speaking, functional differential inclusion system with time-varying and distributed delays can be regard as a generalization of the system described by system with time-varying and distributed delays. Also any mathematical model of the time-varying and distributed delayed dynamic system is the special case of the functional differential inclusion system when uncertainties exist in such a dynamic system.

To best the author's knowledge, only a few papers have studied time-varying and distributed delayed neural networks with discontinuous activations via functional differential inclusions. Under the framework of the theory of Filippov differential inclusions, Liu et al. (2012) has discussed the global convergence of neural networks with mixed time-varying delays and discontinuous activations. He, Lu, and Chen (2009, 2010) investigated the nonnegative periodicity for the neural networks with mixed time-varying delays and discontinuous activations.

Cohen–Grossberg neural networks (CGNN), an important recurrent neural networks model, which was first described by Cohen and Grossberg in 1983 (Cohen & Grossberg, 1983), have aroused a tremendous surge of investigation in these years. However, there is not much work on CGNN discontinuous neuron activations with time delays. Meng, Huang, Guo, and Hu (2010) studied the stability of delayed CGNN with discontinuous neuron activation. Some sufficient conditions are obtained to ensure the