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A new delay-independent condition for global robust stability of neural networks with time delays

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ABSTRACT

This paper studies the problem of robust stability of dynamical neural networks with discrete time delays under the assumptions that the network parameters of the neural system are uncertain and norm-bounded, and the activation functions are slope-bounded. By employing the results of Lyapunov stability theory and matrix theory, new sufficient conditions for the existence, uniqueness and global asymptotic stability of the equilibrium point for delayed neural networks are presented. The results reported in this paper can be easily tested by checking some special properties of symmetric matrices associated with the parameter uncertainties of neural networks. We also present a numerical example to show the effectiveness of the proposed theoretical results.

1. Introduction

In the past years, dynamical neural networks have been used to solve various engineering problems such as pattern recognition, image and signal processing, optimization, associative memory design, control and function approximation. In these classes of engineering applications of neural networks, it is of great importance to know the dynamical behaviors of neural networks. In particular, equilibrium and stability properties of dynamical neural networks play an important role in their design for solving practical problems. For example, when a neural network is aimed to solve some classes of optimization problems, it must be ensured that this neural network has a unique equilibrium point which is globally asymptotically stable. In order to accomplish a targeted problem with neural networks, one has to establish the exact modeling of neural networks. To this end, we have to take the two critical parameters into account: time delays occurring due to the finite speed of information processing and parameter uncertainties due to the existence of external disturbances and parameter deviations. When studying the stability of neural networks, the affect of these parameters must be taken into account as they may cause undesirable dynamical network behaviors such as oscillation and instability. In recent years, global stability of delayed neural networks under parameter uncertainties has been extensively studied and various robust stability conditions for different classes of delayed neural networks have been presented (Arik, 2014a, 2014b; Banu, Balasubramaniam, & Ratnavelu, 2014; Cao, 2001; Cao, Alofi, Mazrooei, & Elaiw, 2013; Cao & Ho, 2005; Cao, Ho, & Huang, 2007; Cao, Huang, & Qu, 2005; Cao, Li, & Han, 2006; Cao & Wan, 2014; Cao & Wang, 2005a, 2005b; Cao, Yuan, & Li, 2006; Ensari & Arik, 2010; Faydasicok & Arik, 2013; Feng, Yang, & Wu, 2015; Guo, Wang, & Yan, 2014; Huang, Feng, & Cao, 2008; Huang, Li, Duan, & Starzyk, 2012; Li, Chen, Zhou, & Qian, 2009; Liang, Wang, Liu, & Liu, 2012; Lou & Cui, 2012; Mathiyalagan, Park, Sakthivel, & Anthoni, 2014; Ozcan & Arik, 2014; Pradeep, Vinodkumar, & Rakkiyappan, 2012; Qiu, Zhang, Wang, Xia, & Shi, 2008; Rakkiyappan, Balasubramaniam, & Krishnasamy, 2011; Shen & Zhang, 2007; Singh, 2007; Wang, Li, & Huang, 2015; Wang, Zhong, Nguang, & Liu, 2013; Wu, Shi, Su, & Chu, 2011; Yang, Gao, & Shi, 2009; Zhang, Liu, & Huang, 2012; Zhang, Ma, Huang, & Wang, 2010; Zhang, Wang, & Liu, 2008, 2009; Zhao, Zhang, Shen, & Gao, 2012; Zhou, Xu, Zhang, Zou, & Shen, 2012; Zhu & Cao, 2010). One of the key factors in the robust stability analysis of neural networks is to find an upper bound for the norm of the intervalized interconnection matrices, and then apply it to the robust stability analysis of neural networks. In some recent papers, four major upper bounds for the norm of the intervalized matrices, are successfully used to derive some sufficient conditions for the robust stability of neural networks. On the other hand, in Arik (2014a), Arik (2014b), Banu et al. (2014), Cao (2001), Cao et al. (2013), Cao and Ho (2005), Cao et al. (2007), Cao et al. (2005), Cao, Li et al. (2006), Cao and Wan (2014), Cao and Wang (2005a), Cao and Wang (2005b), Cao, Yuan et al. (2006), Ensari and Arik (2010), Faydasicok and Arik (2013), Feng et al. (2015), Guo et al. (2014), Huang et al. (2008), Huang et al. (2012), Li et al. (2009), Liang et al.

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(2012), Lou and Cui (2012), Mathiyalagan et al. (2014), Ozcan and Arik (2014), Pradeep et al. (2012), Qiu et al. (2008), Rakkiyappan et al. (2011), Shen and Zhang (2007), Singh (2007), Wang et al. (2015), Wang et al. (2013), Wu et al. (2011), Yang et al. (2009), Zhang et al. (2012), Zhang et al. (2010), Zhang et al. (2008), Zhang et al. (2009), Zhao et al. (2012), Zhou et al. (2012) and Zhu and Cao (2010), some special properties of general matrices have been used to derive alternative robust stability results for neural networks with time delays. Inspired by the results given in Arik (2014a), Arik (2014b), Cao et al. (2005), Ensari and Arik (2010), Faydasicok and Arik (2013), Ozcan and Arik (2014) and Singh (2007), we present a new sufficient condition for robust asymptotic stability of the equilibrium point for the class of delayed neural networks.

This paper is organized as follows. In Section 2, the delayed neural network model with intervalized network parameters is described. In Section 3, sufficient conditions for the global robust asymptotic stability of interval neural networks with time delays are derived by using Lyapunov stability and Homeomorphic mapping theorems with respect to the slope-bounded activation functions. in Section 4, a comparative numerical example is given to demonstrate the effectiveness and applicability of the proposed robust stability condition. Final sections give the concluding remarks.

2. Preliminaries

In this section, we will first give some notations: throughout this paper, the superscript T represents the transpose. I will denote the identity matrix of appropriate dimension. Let $x = (x_1, x_2, \ldots, x_n)^T$. Then, $|x|$ will denote $|x| = (|x_1|, |x_2|, \ldots, |x_n|)^T$. Let $A = (a_{ij})_{n\times n}$ be a real matrix. Then, $|A|$ will denote $|A| = (|a_{ij}|)_{n\times n}$, and $\lambda_m(A)$ and $\lambda_M(A)$ will denote the minimum and maximum eigenvalues of A, respectively. If $A = (a_{ij})_{n\times n}$ is a symmetric matrix, then, $A > 0$ will imply that A is positive definite. We also note the following vector and matrix norms:

$$\|x\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}, \qquad \|A\|_2 = [\lambda_{\max}(A^TA)]^{1/2}.$$

In this paper, we will study the robust stability of the delayed neural network of the following form:

$$\frac{dx_i(t)}{dt} = -c_ix_i(t) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} \times f_j(x_j(t-\tau_j)) + u_i, \quad i = 1, 2, \ldots, n \tag{1}$$

which can be written in matrix–vector form as follows:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau)) + u \tag{2}$$

where n is the number of the neurons, the variables $x_i(t)$ denote the states of the neurons at time t, the functions $f_i(\cdot)$ denote neuron activations, the constant parameters a_{ij} and b_{ij} denote the weight coefficients between neurons j and i at time t and $t-\tau_j$, respectively; the parameters τ_j represent the time delays the constants u_i are the inputs to the neurons, and the constants c_i are the charging rates for the neurons, $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in R^n$ is the state vector, $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $C = \mathrm{diag}(c_i > 0)$, $u = (u_1, u_2, \ldots, u_n)^T \in R^n$, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T \in R^n$ and $f(x(t-\tau)) = (f_1(x_1(t-\tau_1)), f_2(x_2(t-\tau_2)), \ldots, f_n(x_n(t-\tau_n)))^T \in R^n$.

The properties of the activation functions play an important role in determining the stability conditions for neural networks. In this paper, we will assume that the nonlinear activation functions $f_i(\cdot)$ are nondecreasing and slope-bounded. This class of functions is denoted by $f \in \mathcal{K}$ and satisfies the condition that there exist some positive constants k_i such that

$$0 \leq \frac{f_i(x) - f_i(y)}{x - y} \leq k_i, \quad i = 1, 2, \ldots, n, \ \forall x, y \in R, \ x \neq y.$$

Since we aim to study the robust stability of neural system (1), we must formulate the deviations in values of the entries of the interconnection matrices of the neural networks. In general, it is assumed that the perturbations of the network parameters are bounded. Therefore, the matrices $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$ and $C = \mathrm{diag}(c_i > 0)$ of system (1) are defined in the following parameter ranges:

$$\begin{aligned} C_I &= [\underline{C}, \overline{C}] = \{C = \mathrm{diag}(c_i) : 0 < \underline{c}_i \leq c_i \leq \overline{c}_i, \\ &\qquad i = 1, 2, \ldots, n\} \\ A_I &= [\underline{A}, \overline{A}] = \{A = (a_{ij})_{n\times n} : \underline{a}_{ij} \leq a_{ij} \leq \overline{a}_{ij}, \\ &\qquad i, j = 1, 2, \ldots, n\} \\ B_I &= [\underline{B}, \overline{B}] = \{B = (b_{ij})_{n\times n} : \underline{b}_{ij} \leq b_{ij} \leq \overline{b}_{ij}, \\ &\qquad i, j = 1, 2, \ldots, n\}. \end{aligned} \tag{3}$$

We will restate some previous literature results that will play an important role in determining the main result of this paper:

Fact 1 (*Cao et al., 2005*)**.** Let the $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ satisfy (3). Then, there exist positive constants $\sigma(A)$ and $\sigma(B)$ such that

$$\|A\|_2 \leq \sigma(A) \quad \text{and} \quad \|B\|_2 \leq \sigma(B).$$

In the light of Fact 1, in Cao et al. (2005), Ensari and Arik (2010), Faydasicok and Arik (2013) and Singh (2007), various upper bound norms for the interval matrices defined by (3) were introduced. We will unify the results given in Cao et al. (2005) Ensari and Arik (2010), Faydasicok and Arik (2013) and Singh (2007) to state the following lemma:

Lemma 1. *Consider a real matrix B defined by $B \in B_I = [\underline{B}, \overline{B}] = \{B = (b_{ij})_{n\times n} : \underline{b}_{ij} \leq b_{ij} \leq \overline{b}_{ij},\ i, j = 1, 2, \ldots, n\}$. Define $B^* = \frac{1}{2}(\overline{B} + \underline{B})$, $B_* = \frac{1}{2}(\overline{B} - \underline{B})$ and $\hat{B} = (\hat{b}_{ij})_{n\times n}$ with $\hat{b}_{ij} = \max\{|\underline{b}_{ij}|, |\overline{b}_{ij}|\}$. Let*

$$\sigma_1(B) = \sqrt{\|\, |B^{*T}B^*| + 2|B^{*T}|B_* + B_*^TB_*\|_2}$$

$$\sigma_2(B) = \|B^*\|_2 + \|B_*\|_2$$

$$\sigma_3(B) = \sqrt{\|B^*\|_2^2 + \|B_*\|_2^2 + 2\|B_*^T|B^*|\,\|_2}$$

$$\sigma_4(B) = \|\hat{B}\|_2.$$

Then, the following inequality holds:

$$\|B\|_2 \leq \sigma_m(B)$$

where $\sigma_m(B) = \min(\sigma_1(B), \sigma_2(B), \sigma_3(B), \sigma_4(B))$.

It should be pointed out here that there are no direct relationships among $\sigma_1(B), \sigma_2(B), \sigma_3(B), \sigma_4(B)$. It is possible to establish four different cases where in each case $\sigma_i(B) = \sigma_m(B)$, for $i = 1, 2, 3, 4$.

Lemma 2 (*Arik, 2014b*)**.** *If A is a real matrix defined by $A \in A_I = [\underline{A}, \overline{A}] = \{A = (a_{ij})_{n\times n} : \underline{a}_{ij} \leq a_{ij} \leq \overline{a}_{ij},\ i, j = 1, 2, \ldots, n\}$, then, for $x = (x_1, x_2, \ldots, x_n)^T$, there exist a positive diagonal matrix P and a nonnegative diagonal matrix Υ such that the following inequality holds:*

$$x^T(PA + A^TP)x \leq x^T(P(A^* - \Upsilon) + (A^* - \Upsilon)^TP + \|P(A_* + \Upsilon) + (A_* + \Upsilon)^TP\|_2)x$$

where $A^* = \frac{1}{2}(\overline{A} + \underline{A})$, $A_* = \frac{1}{2}(\overline{A} - \underline{A})$.