Neural Networks 43 (2013) 99–113

Contents lists available at SciVerse ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

# A supervised multi-spike learning algorithm based on gradient descent for spiking neural networks

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ARTICLE INFO

*Article history:*
Received 4 August 2011
Received in revised form 5 February 2013
Accepted 8 February 2013

*Keywords:*
Spiking neural networks
Multi-spike learning
Single-spike learning
Spike sequence learning
Classification

ABSTRACT

We use a supervised multi-spike learning algorithm for spiking neural networks (SNNs) with temporal encoding to simulate the learning mechanism of biological neurons in which the SNN output spike trains are encoded by firing times. We first analyze why existing gradient-descent-based learning methods for SNNs have difficulty in achieving multi-spike learning. We then propose a new multi-spike learning method for SNNs based on gradient descent that solves the problems of error function construction and interference among multiple output spikes during learning. The method could be widely applied to single spiking neurons to learn desired output spike trains and to multilayer SNNs to solve classification problems. By overcoming learning interference among multiple spikes, our method has high learning accuracy when there are a relatively large number of output spikes in need of learning. We also develop an output encoding strategy with respect to multiple spikes for classification problems. This effectively improves the classification accuracy of multi-spike learning compared to that of single-spike learning.


## 1. Introduction

With the aim of achieving artificial intelligence, artificial neural networks are used to structurally simulate biological nervous systems and the functions of the human brain. Over recent decades, the study of artificial neural networks has developed from the simple to the complex. Perceptron (Minsky & Papert, 1969), the simplest artificial neuron model, is a highly abstract and simplified model of the way in which a biological neuron works. Its activation function, the hard limit function, is very simple, which restricts its learning ability and working functionality. Progress in artificial neural networks has involved the use of continuous and differentiable functions as activation functions with the BP algorithm (Rumelhart, Hinton, & Williams, 1986). However, these artificial neuron models are still much simpler than biological neurons. Therefore, spiking neurons and SNNs whose operating mode is closer to biological neurons were proposed (Maass, 1997b) and have been studied widely.

In biological nervous systems, a neuron transmits information to others by spike trains with a specific frequency and amplitude (Bose & Liang, 1996; Gerstner & Kistler, 2002). The information is usually encoded in two ways: the spiking frequency (*rate encoding*) (Adrian & Zotterman, 1926; Kandel, Schwartz, & Jessel, 1991), and the spike firing time (*pulse or temporal encoding*) (Rullen, Guyonneau, & Thorpe, 2005; Rullen & Thorpe, 2001; Theunissen & Miller, 1995). The input and output of the sigmoidal neurons with continuous and differentiable functions are both real numbers and can be supposed to be similar to the rate encoding. However, use of only rate encoding will result in loss of information in the form of precise spike firing times. Spiking neurons whose input and output are spike firing times can compensate for this disadvantage. The spike train itself, which is encoded by the precise spike firing times, contains information on the spiking rate, so spiking neurons are better than sigmoidal neurons in terms of function. In fact, SNNs can simulate any sigmoidal neural networks (Maass, 1997a). Furthermore, it has been shown that networks of noisy spiking neurons with temporal encoding have strictly greater computational power than sigmoidal neural networks with the same number of neurons (Maass, 1997c).

Supervised learning based on temporal encoding of spikes is an important part of SNN research since the way in which a spiking neuron works is similar to that of a biological neuron. However, the exact mechanism of supervised learning in biological neurons remains unclear (Ponulak & Kasiński, 2010). Furthermore, supervised learning methods for perceptron and sigmoidal neural networks cannot be directly applied to SNNs. Therefore, research into supervised learning for SNNs is still at the primary stage and existing learning methods (Belatreche, Maguire, McGinnity, & Wu, 2003; Carnell & Richardson, 2005; Gütig & Sompolinsky, 2006; Pfister, Toyoizumi, Barber, & Gerstner, 2006; Schrauwen & Campenhout, 2006) have some weaknesses (Kasiński & Ponulak, 2006; Ponulak & Kasiński, 2010). At present, according to the

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doi:10.1016/j.neunet.2013.02.003

number of spikes during learning, supervised learning based on temporal encoding can be classified into two types: single-spike and multi-spike learning (spike sequence learning).

The first research was for single-spike learning. The most typical methods are SpikeProp and its various improvements, which are based on gradient descent. Owing to the discrete feature of spikes and to overcome the lack of continuous and differentiable activation functions in spiking neurons, Bohte, Kok and La Poutré (2002) proposed the SpikeProp learning algorithm for feedforward multilayer SNNs and successfully applied it to classification problems. SpikeProp makes error back-propagation available by assuming that the value of the internal state of a neuron increases linearly in the infinitesimal time around the instant of neuronal firing. On this basis, various learning algorithms such as back-propagation with momentum, QuickProp, resilient propagation (RProp), and Levenberg–Marquardt BP (Ghosh-Dastidar & Adeli, 2007; McKennoch, Liu, & Bushnell, 2006; Schrauwen & Campenhout, 2004; Silva & Ruano, 2005; Xin & Embrechts, 2001) were proposed to improve the performance of SNN training. All of the above methods are suitable for networks with an absolute single-spiking structure, that is, neurons in the input, output and hidden layers can only emit a single spike. Booij and Nguyen (2005) first proposed a back-propagating learning model that can output multiple spikes. However, only neurons in the hidden layers are allowed to emit multiple spikes in this model, and only the first output spike of neurons in the output layer can be controlled through training. Similarly, Ghosh-Dastidar and Adeli (2009) proposed a multi-spiking network model called MuSpiNN with a back-propagating learning algorithm named Multi-SpikeProp. Multiple spikes are allowed in the hidden layers of MuSpiNN, but only one output spike of each neuron in the output layer is used to construct the error function. Therefore, the above two learning methods are still single-spike learning models. Moreover, the error back-propagation method for single-spike learning is applicable not only to feedforward SNNs but also to recurrent SNNs (Tiňo & Mills, 2006).

In multi-spike learning, the times of multiple output spikes need to be controlled during a relatively long run time for SNNs, which is more complicated than in single-spike learning. Almost all existing multi-spike learning methods focus on single spiking neurons or single-layer SNNs, which contain many presynaptic inputs and learn to output determinate spike trains. For example, Carnell and Richardson (2005) proposed a spike sequence learning method that uses linear algebraic means. Although this method is effective, it lacks biological explanation. To better simulate biological neurons, most researchers focus on supervised learning methods with biological meaning. Spike-timing-dependent plasticity (STDP) (Bi & Poo, 1998; Markram, Lübke, Frotscher, & Sakmann, 1997) is a learning mode obtained from biological experiments. Although STDP can only run under an unsupervised mode, its theory and principles appear in many existing supervised learning methods (Legenstein, Naeger, & Maass, 2005; Pfister et al., 2006). Some learning methods combine reinforcement learning with STDP to realize supervised learning based on rate or temporal encoding (Florian, 2007; Legenstein, Pecevski, & Maass, 2008). Ponulak (2005) proposed a supervised learning algorithm for a spiking neuron called Remote Supervised Method (ReSuMe) and later provided detailed analyses of the algorithm (Ponulak & Kasiński, 2010). In the learning process of ReSuMe, the desired (supervisory) signal does not directly influence the membrane potential of the corresponding learning neuron. More specifically, the basic of ReSuMe is a Widrow–Hoff rule, from which, the practical learning algorithm composed of two weight update processes is derived. These are an STDP process for strengthening synapses based on input spike trains and the desired output spike train, and an anti-STDP process (Bi & Poo, 1998; Kistler, 2002) for weakening synapses based on the input spike trains and the actual output spike train. Results show that ReSuMe has good learning ability and wide applications.

The above is a brief introduction to existing supervised learning methods for spiking neurons and SNNs. Readers are referred to (Kasiński & Ponulak, 2006) for more detailed information. From the introduction above it is evident that among the supervised learning methods based on temporal encoding, the learning objectives of single-spike learning are relatively simple, so single-spike learning can easily be applied to multilayer feedforward or feedback SNNs. Owing to the intricacy of the learning objectives, most multi-spike learning methods can only be applied to single neurons or single-layer SNNs. Thus, multi-spike learning methods for multilayer SNNs require further investigation.

Spike trains with temporal encoding or firing times of multiple spikes carry important information in biological neurons and this information cannot be expressed by rate encoding or simple single-spike forms. Furthermore, to the best of our knowledge there is no method that can transform multi-spike learning into rate encoding and single-spike learnings. Consideration of only single-spike learning is inconsistent with biological basis and will lead to the ability shortage of SNNs to express information. To simulate the activity of biological nervous systems, it is necessary to study multi-spike learning for SNNs. Real biological nervous systems are extremely complex networks composed of a large number of interconnected neurons, thus single neurons and single-layer SNNs are too simple to sufficiently simulate the learning of complex biological nervous systems. It is necessary to study multilayer learning for SNNs, which is the basis for research on the learning of more complex SNNs. Therefore, research on multi-spike learning for multilayer SNNs is very significant.

An advantage of gradient-descent-based (GDB) supervised learning algorithms such as SpikeProp is easy realization of learning for multilayer SNNs. There are two main reasons why researchers have not researched multi-spike learning in this field. First, the number of actual output spikes may not be the same as the number of desired output spikes in the presence of multiple spikes, so construction of the error function is complicated. Second, learning interference of weight adjustment among multiple spikes (i.e. the weight adjustment when a neuron learns one spike) will affect the neurons that have already learned from other spikes. In this study we analyze the two difficulties of multi-spike learning, identify theoretical bases and methods to solve the problems, and propose a multi-spike learning algorithm based on gradient descent.

The method proposed here has good applicability. It can be applied not only to single spiking neurons but also to multilayer SNNs through back-propagation. For spike-sequence learning for single neurons, the performance of a learning method gradually decreases as the number of desired output spikes increases. Because our solution to overcome learning interference among multiple spikes is put forward, the performance of our method decreases less than other methods such as ReSuMe. In other words, when the number of desired output spikes is large, our method has the advantage of high learning accuracy, which can be verified by experiments. When using multi-spike learning for multilayer SNNs to solve classification problems, more desired output targets can be learned during the learning process. Therefore, the classification performance of multi-spike learning should be better than single-spike learning theoretically. However, we find in experiments that if the desired multiple output spikes representing different classes are arbitrarily set, the classification performance of multi-spike learning is weaker than that of single-spike learning. We analyze the reason for this phenomenon and propose a new encoding strategy for desired multiple output spikes. The classification performance of multi-spike learning can be effectively improved by adopting this encoding strategy.

The remainder of the paper is organized as follows. In Section 2 we define a spiking neuron and the SNN model used. In Section 3