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Global exponential estimates of delayed stochastic neural networks with Markovian switching

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ABSTRACT

This paper is concerned with the global exponential estimating problem of delayed stochastic neural networks with Markovian switching. By fully taking the inherent characteristic of such kinds of neural networks into account, a novel stochastic Lyapunov functional is constructed in which as many as possible of the positive definite matrices are dependent on the system mode and a triple-integral term is introduced. Based on it, a delay- and mode-dependent criterion is derived under which not only the neural network is mean square exponentially stable but also the decay rate is well obtained. Moreover, it is shown that the established stability condition includes some existing ones as its special cases, and is thus less conservative. This approach is then extended to two more general cases where mode-dependent time-varying delays and parameter uncertainties are considered. Finally, three numerical examples are presented to demonstrate the performance and effectiveness of the developed approach.

1. Introduction

During the past few years, various kinds of recurrent neural networks have been proposed including bidirectional associative memory neural networks, cellular neural networks, Cohen–Grossberg neural networks and Hopfield neural networks, etc. Many exciting applications have been established in knowledge acquisition, combinatorial optimization, adaptive control, signal processing, prediction and other areas (Haykin, 1999). Generally, a prerequisite to these successful applications is the stability of the underlying neural networks. As a result, much effort has been devoted to the stability analysis of recurrent neural networks.

In electronic implementations of neural networks, time delays are frequently inevitable in the process of information storage and transmission. A main disadvantage of the presence of time delays is to lead to instability and oscillation. On the other hand, it has been recognized that better performance can be achieved when time delays are intentionally introduced for some special circumstances (Roska & Chua, 1992) (e.g., speed detection of moving objects and processing of moving images). Consequently, the study of recurrent neural networks with time delays has gained a great deal of attention. Many interesting results related to the stability analysis have been reported in the literature (Faydasicok & Arik, 2012; Li, Gao, & Yu, 2011; Liu, Chen, Cao, & Lu, 2011; Marco, Grazzini, & Pancioni, 2011; Wu, Liu, Shi, He, & Yokoyama, 2008; Zeng & Wang, 2006; Zhang & Han, 2009; Zhang, Liu, Huang, & Wang, 2010; Zheng, Zhang, & Wang, 2011).

In real nervous systems, the synaptic transmission can be regarded as a noisy process because of random fluctuations from the release of neurotransmitters and other probabilistic causes. As observed in Blythe, Mao, and Shah (2001) and Liao and Mao (1996); Shen and Wang (2007), a neural network can be stabilized or destabilized by certain stochastic inputs. It motivates the study of the stability analysis problem of stochastic neural networks (see, for examples, Chen & Zheng, 2010, Wang, Liu, Li, & Liu, 2006, Yang, Gao, & Shi, 2009, Zhang, Xu, Zong, & Zou, 2009 and the references therein).

Furthermore, in practice, the phenomenon of information latching often appears in neural networks. Fortunately, it can be efficiently tackled by extracting a finite state representation from the network (Tino, Cernansky, & Benuskova, 2004; Wu, Shi, Su, & Chu, 2011). That is to say, the neural networks with information latching may have finite modes which can switch from one to another at different times. It has been known (Zhang & Wang, 2008; Zhu & Cao, 2011) that the Markov chain provides one of the promised ways to characterize the switching between different modes. Therefore, the study of delayed stochastic neural networks with Markovian switching is of great significance and practical importance, and plays an essential role to the potential applications in the field of information science. Recently, many methods (for examples, the delay partitioning technique and the free-weighting

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matrices based method) have been adopted to deal with this issue (Balasubramaniam & Lakshmanan, 2009; Huang, Ho, & Qu, 2007; Liu, Ou, Hu, & Liu, 2010; Liu, Wang, Liang, & Liu, 2009; Liu, Wang, & Liu, 2008; Lou & Cui, 2007; Ma, Xu, Zou, & Lu, 2011; Wang, Liu, Yu, & Liu, 2006; Wu, Shi, Su, & Chu, 2012; Yang, Cao, & Lu, 2012; Zhang & Wang, 2008; Zhu & Cao, 2010, 2011). In Wang, Liu, Yu et al. (2006), the authors discussed the exponential stability analysis problem of recurrent neural networks with time delays and Markovian jumping parameters. A delay-independent condition was obtained by means of linear matrix inequalities (LMIs) (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994). At the same time, the authors in Huang et al. (2007) studied the global robust stability of stochastic additive neural networks with Markovian switching and interval uncertainties. In Ma et al. (2011), a delay-dependent stability condition was derived for uncertain stochastic neural networks with Markovian jumping parameters and mixed mode-dependent delays by introducing some slack matrices (or free-weighting matrices). In Liu et al. (2010), the authors studied the stability analysis problem of delayed bidirectional associative memory neural networks with Markovian jumping parameters by using the delay partitioning approach. However, it should be noted that, in most of the above results, only a part of the positive definite matrices (i.e., the matrices involved in the quadratic form and the single-integral terms of the constructed stochastic Lyapunov functionals) are dependent on the system mode, while the matrices in the double-integral terms are common for all modes. It is thus expected that less conservative stability criteria could be established if more positive definite matrices are chosen to be mode-dependent. This is because the choice of the positive definite matrices in the latter case obviously has more freedom than that in the former case. In addition, as suggested in Shu and Lam (2008), the transient process of a neural network can be clearly characterized once its decay rate is explicitly known. Therefore, the exponential stability analysis is also of practical value. These motivate the present study.

In this paper, our attention focuses on the global exponential estimating problem of a class of delayed stochastic neural networks with Markovian switching. By fully considering its inherent characteristic, a new stochastic Lyapunov functional is constructed with as many as possible mode-dependent positive definite matrices and an additional tripe-integral term. The role of this triple-integral term is such that some positive definite matrices in the double-integral terms, which are common in the above literature, depend on the system mode. Then, a delay- and mode-dependent stability condition is established in terms of LMIs. It should be pointed out that the obtained LMIs are monotonically increasing with respect to the decay rate. Therefore, the upper bound of the decay rate can be efficiently found by solving a corresponding convex optimization problem, which is facilitated readily by some available algorithms (e.g., the interior point algorithm) (Boyd et al., 1994). It is further shown from a point of view of theory that the stability criterion includes some previous ones as its special cases and is thus less conservative. The main contributions of this study are that (i) a novel stability condition is derived; (ii) the upper bound of the decay rate can be easily obtained; and (iii) the less conservatism of our approach is rigorously proven. Moreover, this approach is then extended to address the global exponential estimating problem of stochastic neural networks with Markovian switching, mode-dependent time-varying delays and parameter uncertainties. Finally, several examples are provided to illustrate the performance and effectiveness of the developed approach.

Notation: The following notations will be used throughout this paper. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^+$ the set of nonnegative real numbers, $\mathbb{R}^n$ the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ the set of all $n \times m$ real matrices. The superscript "T" represents the transpose. I is the identity matrix with appropriate dimension. The symbol $*$ denotes the symmetric block in a symmetric matrix. For any real square matrices X and Y, $X > Y$ ($X \geq Y, X < Y, X \leq Y$) means that X–Y is symmetric and positive definite (positive semi-definite, negative definite, negative semi-definite, respectively), and $X > 0$ ($X \geq 0, X < 0, X \leq 0$) means that X is symmetric and positive definite (positive semi-definite, negative definite, negative semi-definite, respectively); $\mathrm{Tr}(X)$ is the trace of X; $\lambda_{\max}(X)$ and $\lambda_{\min}(X)$ are respectively the maximum and minimum eigenvalues of X. For $\tau > 0$, $\mathscr{C}([-\tau, 0]; \mathbb{R}^n)$ denotes the family of continuous functions φ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-\tau\leq\vartheta\leq 0} |\varphi(\vartheta)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. Let $(\Omega, \mathscr{F}, \mathscr{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathscr{F}_0$ contains all $\mathscr{P}$-pull sets); $\mathscr{C}^b_{\mathscr{F}_0}([-\tau, 0]; \mathbb{R}^n)$ the family of all bounded, $\mathscr{F}_0$-measurable, $\mathscr{C}([-\tau, 0]; \mathbb{R}^n)$-valued random variables; $\mathscr{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}^+ \times S; \mathbb{R}^+)$ the family of all nonnegative functions $V(u, t, i)$ on $\mathbb{R}^n \times \mathbb{R}^+ \times S$ which are continuously twice differentiable in u and differentiable in t. The mathematical expectation operator with respect to a given probability measure $\mathscr{P}$ is denoted by $\mathbb{E}$.

2. Problem formulation

Similar to (Huang et al., 2007; Wang, Liu, Yu et al., 2006; Zhu & Cao, 2011), the delayed stochastic neural network with Markovian switching considered in this paper is described by

$$\begin{aligned} du(t) &= [-E(r(t))u(t) + A(r(t))f(u(t)) \\ &\quad + C(r(t))h(u(t-\tau(t)))]dt \\ &\quad + \sigma(u(t), u(t-\tau(t)), t, r(t))dw(t) \end{aligned} \tag{1}$$

$$u(t) = \xi(t), \quad t \in [-\tau, 0],\ r(0) = r_0, \tag{2}$$

where $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T$ is the state vector associated with n neurons, $w(t)$ is a m-dimensional Browian motion on the complete probability space $(\Omega, \mathscr{F}, \mathscr{P})$, $\{r(t)\}_{t\geq 0}$, which is supposed to be independent of $w(t)$, is a right-continuous Markov chain defined on the complete probability space $(\Omega, \mathscr{F}, \mathscr{P})$ and taking values in a finite state space $S = \{1, 2, \ldots, N\}$ with transition probability matrix $Q = (q_{ij})_{N\times N}$ given by

$$\mathscr{P}\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} q_{ij}\Delta + o(\Delta), & \text{if } i \neq j \\ 1 + q_{ii}\Delta + o(\Delta), & \text{if } i = j \end{cases}$$

with $\Delta > 0$ and $\lim_{\Delta\to 0+} o(\Delta)/\Delta = 0$. Here, $q_{ij} \geq 0$ ($i \neq j$) is the transition rate from i to j, and

$$q_{ii} = -\sum_{j=1, j\neq i}^{N} q_{ij}. \tag{3}$$

$E(r(t)) = \mathrm{diag}(e_1(r(t)), e_2(r(t)), \ldots, e_n(r(t)))$ is the firing rate matrix with positive entries, $A(r(t))$ and $C(r(t))$ are respectively the connection weight matrix and the delayed connection weight matrix, $f(u(t)) = [f_1(u_1(t)), f_2(u_2(t)), \ldots, f_n(u_n(t))]^T$ and $h(u(t)) = [h_1(u_1(t)), h_2(u_2(t)), \ldots, h_n(u_n(t))]^T$ are the activation functions, the noise perturbation $\sigma : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^+ \times S \to \mathbb{R}^{n\times m}$ is a Borel measurable function, $\tau(t)$ is a time-varying delay with an upper bound $\tau > 0$, $\xi(t) \in \mathscr{C}^b_{\mathscr{F}_0}([-\tau, 0]; \mathbb{R}^n)$ is an initial function and $r_0 \in S$ is an initial mode.

For the sake of simplicity, for each $r(t) = i \in S$, we denote $E(r(t)) = E_i, A(r(t)) = A_i, C(r(t)) = C_i$ and $\sigma(u(t), u(t-\tau(t)), t, r(t)) = \sigma(u(t), u(t-\tau(t)), t, i)$ (or sometimes $\sigma(t, i)$). As in Zhu and Cao (2011), the following assumptions are always made: