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Exponential stability of delayed and impulsive cellular neural networks with partially Lipschitz continuous activation functions

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ABSTRACT

The paper discusses exponential stability of distributed delayed and impulsive cellular neural networks with partially Lipschitz continuous activation functions. By relative nonlinear measure method, some novel criteria are obtained for the uniqueness and exponential stability of the equilibrium point. Our method abandons usual assumptions on global Lipschitz continuity, boundedness and monotonicity of activation functions. Our results are generalization and improvement of some existing ones. Finally, two examples and their simulations are presented to illustrate the correctness of our analysis.

1. Introduction

Cellular neural networks (CNNs) initially introduced by Chua and Yang (1988a, 1988b) have found many important applications in the solving optimization problem, pattern recognition, biology and image processing, especially in static image treatment (Civalleri, Gilli, & Pandolfi, 1993). From a mathematical point of view, a CNN can be characterized by an array of identical nonlinear dynamical systems (called cells) that are locally interconnected as in Gilli, Biey, and Checco (2004), which presented a set of sufficient conditions ensuring the existence of at least one stable equilibrium point in terms of the template elements.

In electronic implementation of neural networks, time delays are inevitable due to axonal conduction times and finite switching speeds of amplifiers (Chen & Zheng, 2010). Moreover, processing of moving images requires the introduction of delays in the signals transmitted among the cells (Roska & Chua, 1990). However, time delays may destroy stability of the networks and even lead to the oscillation behaviors. Consequently, it is required to study the stability of CNNs with delays and there come forth many excellent results on stability of CNNs with discrete delays (Chen & Zheng, 2010; Civalleri et al., 1993; Gilli, 1994; He, Wu, & She, 2006; Hu, Gao, & Zheng, 2008; Liu & Cao, 2006; Park, 2006; Xiao & Zhang, 2009; Xu, Lam, Ho, & Zou, 2005; Zeng & Wang, 2006; Zhang & Gui, 2009; Zhang & Wang, 2007; Zhang, Wei, & Xu, 2005; Zhao & Cao, 2005). Obviously, it is not reasonable that time delays are assumed to be discrete because the time delays are continuously distributed over a certain duration of time such that the distant past has less influence compared to the recent behavior of the state. The duration over which the past effects affect the current state can extend over a finite or infinite interval (Mohamad, 2007). To overcome the shortcoming, distributed delays proposed first by Gopalsamy (1992) were introduced into the model of CNNs (Liao, Wu, & Yu, 2002; Liu, You, & Cao, 2007; Ma, Yu, & Zhang, 2009; Mohamad, 2007). Moreover, the abrupt changes in the voltages produced by faulty circuit elements are exemplary of impulse phenomena which can affect the transient behavior of the network (Ahmada & Stamovab, 2008). A great deal of attention has been devoted to stability analysis of different types of impulsive CNNs with discrete delays (Li, Hua, & Fei, 2009; Stamova & Ilarionov, 2010; Xia, Cao, & Cheng, 2007; Xia, Huang, & Han, 2008; Yang, Cui, & Long, 2009; Zhang, 2009) and with distributed delays (Ahmada & Stamovab, 2008; Feng & Lam, 2011; Huang, Luo, & Yang, 2007; Kaslik & Sivasundaram, 2011; Li & Yang, 2006; Li, Zhang, & Li, 2009; Liu & Huang, 2006; Mohamad, Gopalsamy, & Akca, 2008; Ping & Lu, 2009; Wang et al., 2006; Yin & Li, 2009; Zhou, 2009).

According to foregoing analysis, it is more reasonable to discuss the stability of CNNs with impulse and distributed delays. Among the existing research results, some activation functions are assumed to be globally Lipschitz continuous (Li & Yang, 2006; Li, Zhang et al., 2009; Liu & Huang, 2006; Mohamad et al., 2008;

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Ping & Lu, 2009; Wang et al., 2006; Zhou, 2009), bounded and monotonic (Huang et al., 2007) and bounded (Ahmada & Stamovab, 2008; Kaslik & Sivasundaram, 2011; Yin & Li, 2009). Unfortunately, these assumptions make these existing results unapplicable to some important engineering problems. For example, when the neural networks are used to solve optimization problems with the presence of constraints (linear, quadratic, or more general programming problems), unbounded (or nonmonotonic, non-globally Lipschitz continuous) activations modeled by diode-like exponential-type functions are needed such that constraints are satisfied (Forti & Tesi, 1995). Motivated by this, we attempt to abandon these assumptions and only require activation functions to be partially Lipschitz continuous. Moreover, the relative nonlinear measure is more efficient than the nonlinear measure for exponential stability analysis of Hopfield-type neural networks without delays where the equilibrium points are given (Qiao, Peng, & Xu, 2001). Consequently, the paper is devoted to studying the exponential stability of distributed delays and impulsive CNNs with continuous Lipschitz continuous activation functions by relative nonlinear measure.

The remainder of this paper is arranged as follows. In Section 2, the original state equation of CNNs is reformed as a nonlinear differential system with distributed delays. In Section 3, being preliminaries, a nonlinear impulsive functional differential system with distributed delays is discussed and sufficient conditions are presented for exponential stability of equilibrium point of the system. In Section 4, some sufficient conditions are provided for exponential stability of equilibrium point of different types of CNNs by means of results derived in Section 3. Moreover, two examples and their simulations are presented to illustrate that our method is valid and that our derived results are new and correct. Conclusions are given in Section 5.

2. Problem reformulation

The state equation for the cell $C(i,j)$ of an $M \times N$ CNNs in Chua and Yang (1988a, 1988b) is described by the following

$$C_x \frac{dv_{xij}(t)}{dt} = -\frac{1}{R_x} v_{xij}(t) + \sum_{C(k,l)\in N_r(i,j)} A(i,j;k,l) v_{ykl}(t) + \sum_{C(k,l)\in N_r(i,j)} B(i,j;k,l) v_{ukl} + I_{ij},$$
$$1 \le i \le M,\ 1 \le j \le N, \tag{1}$$

where v_{xij}, v_{uij} and v_{yij} denote the state voltage, input voltage and output voltage of a cell respectively; $A(i,j;k,l)$ and $B(i,j;k,l)$ are the linear feedback and control operators, respectively; I_{ij} is an independent bias current source; $N_r(i,j) = \{C(k,l) \mid \max\{|k-i|, |l-j|\} \le r, 1 \le k \le M; 1 \le l \le N\}$ (r is a positive integer number); The output is defined by $v_{yij} = f(v_{xij})$ where $f(x) = 0.5(|x+1| - |x-1|)$.

After having ordered the cells in some way (e.g., by columns or rows), the state equation of CNNs (1) composed of $M \times N$ cells can be reformed in Civalleri et al. (1993) as the following vector form

$$\dot{x} = -x + \hat{A}x + \hat{B}u + I, \tag{2}$$

where $x, \dot{x} \in \mathbb{R}^{M\times N}$ denote state vector and its derivative, respectively; $y \in \mathbb{R}^{M\times N}$ is output vector depending on x through the saturation function defined in Chua and Yang (1988a); $u \in \mathbb{R}^{M\times N}$ is input vector; $I \in \mathbb{R}^{M\times N}$ is the bias current vector; Both the linear cell resistance R_x and capacitance C_x in (1) are assumed to be 1; $\hat{A}, \hat{B} \in \mathbb{R}^{M\times N, M\times N}$ depend on the established order among the cells and on the cloning templates.

To discuss effect of delays on the CNNs, the paper Civalleri et al. (1993) further investigated the modified state equation with the following vector form:

$$\begin{aligned}\dot{x}(t) &= -x(t) + Ay(t) + A^\tau y(t-\tau) + Bu(t) + I \\ &= -x(t) + Af(x(t)) + A^\tau f(x(t-\tau)) + Bu(t) + I,\end{aligned} \tag{3}$$

where

$$x(t) = \{v_{xij}(t)\} \quad i = 1,2,\ldots,M; j = 1,2,\ldots,N,$$
$$y(t) = \{v_{ykl}(t)\} \quad k = 1,2,\ldots,M; l = 1,2,\ldots,N,$$
$$u(t) = \{v_{ukl}(t)\} \quad k = 1,2,\ldots,M; l = 1,2,\ldots,N,$$
$$I = \{I_{ij}\} \quad i = 1,2,\ldots,M; j = 1,2,\ldots,N,$$

denotes the cell voltage, output/input node voltage and external biasing current vectors, respectively; A, A^τ and B are the feedback template, delayed template and control template, respectively. Both the linear cell resistance R_x and capacitance C_x in (1) are assumed to be 1; the time delay τ is assumed to be discrete values, i.e., $\tau \in [0, \infty)$ and f is the activation function of the neuron cells.

Owing to the fact that discrete delays cannot characterize the delays caused by propagation on a multitude of parallel pathways with a variety of axon sizes and lengths, the delayed output voltage $y(t-\tau) = f(x(t-\tau))$ in (3) should be replaced by a more general expression defined below

$$\int_{-\infty}^{t} K(t-s) f(x(s)) ds \quad \text{or} \quad f\left(\int_{-\infty}^{t} K(t-s)(x(s)ds)\right),$$

where $K(t-s) = \{K_{ij}(t-s)\}(i = 1,2,\ldots,M; j = 1,2,\ldots,N)$ is a Kernel vector characterizing the refractoriness of the neuron cells (Liao et al., 2002).

In this paper, we plan to discuss the model of an $M \times N$ CNNs with distributed delays and impulses described by the following differential equations

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -b_i x_i(t) + \sum_{j=1}^{p} a_{ij} f_j(v_j x_j(t)) \\ &\quad + \sum_{j=1}^{p} a_{ij}^\tau \int_{-\infty}^{t} K_{ij}(t-s) g_j(v_j^\tau x_j(s)) ds \\ &\quad + \sum_{j=1}^{p} B_{ij} u_j + I_i, \quad t \ge 0,\ t \ne t_k\end{aligned} \tag{4}$$
$$\Delta x_i(t_k) = I_i(x_i(t_k)), \quad i = 1,2,\ldots,p,\ k \in \mathbb{N},$$

where $p = M \times N$ and $i,j = 1,2,\ldots,p$; $b_i = \frac{1}{R_x C_x}$ is a positive constant and represents the rate with which the ith cell will reset its potential to the resting state in isolation when disconnected from the network and external inputs; v_j and v_j^τ denote the normal and the delay amplifier gain of the jth cell, respectively; $A = (a_{ij})_{p\times p}$, $A^\tau = (a_{ij}^\tau)_{p\times p}$ and $B = (B_{ij})_{p\times p}$ are the feedback template, delayed template and control template, respectively. $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$ is the impulse at moments t_k and $t_1 < t_2 < \cdots$ is a strictly increasing sequences such that $\lim_{k\to+\infty} t_k = +\infty$; f and g are activation functions of the neuron cells. The initial condition associated with the model (4) satisfies

$$u_i = \phi_i \in C((-\infty, 0], \mathbb{R}), \quad i = 1,2,\ldots,p,$$

where $C((-\infty,0],\mathbb{R})$ denotes the set of all bounded continuous functions from $(-\infty, 0]$ to real number space $\mathbb{R}$.

In order to investigate the stability of the model (4), we only suppose that

(H_1) Activation functions f_j and g_j are partially Lipschitz continuous for $j = 1,2,\ldots,p$;