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Dissipativity and quasi-synchronization for neural networks with discontinuous activations and parameter mismatches

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ABSTRACT

In this paper, global dissipativity and quasi-synchronization issues are investigated for the delayed neural networks with discontinuous activation functions. Under the framework of Filippov solutions, the existence and dissipativity of solutions can be guaranteed by the matrix measure approach and the new obtained generalized Halanay inequalities. Then, for the discontinuous master–response systems with parameter mismatches, quasi-synchronization criteria are obtained by using feedback control. Furthermore, when the proper approximate functions are selected, the complete synchronization can be discussed as a special case that two systems are identical. Numerical simulations on the chaotic systems are presented to demonstrate the effectiveness of the theoretical results.

1. Introduction

Neural networks with discontinuous (or non-Lipschitz, or non-smooth) neuron activations, have been found useful to address a number of interesting engineering tasks, such as dry friction, impacting machines, systems oscillating under the effect of an earthquake, power circuits, switching in electronic circuits and many others, and therefore have received a great deal of attention in the literature (Cortés, 2008; Danca, 2002; Forti & Nistri, 2003; Forti, Nistri, & Papini, 2005; Liu & Cao, 2009; Lu & Chen, 2006, 2008). In linear and nonlinear programming, the discontinuous neural networks (DNNs) are able to execute the circuit equilibrium points coinciding with the constrained critical points of the objective function (Chong, Hui, & Zak, 1999; Ferreira, Kaszkurewicz, & Bhaya, 2005; Forti & Tesi, 1995). The best property of such networks that should be stressed is the global convergence in finite time, in comparison to smooth dynamical systems which can only converge as time goes to infinity. Such a property seems especially important in a global optimization problem since the minimum can be computed in real time (Forti & Nistri, 2003; Wang & Xiao, 2010).

In the literature of analyzing DNNs, fundamental results have been established on (robust) stability or convergence of the equilibrium point or periodic solutions for delayed Hopfield DNNs (Forti & Nistri, 2003; Forti et al., 2005; Liu & Cao, 2009) and Cohen–Grossberg DNNs (Lu & Chen, 2006, 2008). The stability problem of an equilibrium point is indeed central to the analysis of a dynamic system. Nevertheless, from a practical point of view, it is not always the case that the orbits of the neural network approach a single equilibrium point. It is possible that there is no equilibrium point in some situations. Therefore, the concept on dissipativity was introduced (Cao, Yuan, Ho, & Lam, 2006; Hale, 1989; Liao & Wang, 2003; Song & Cao, 2008) and has applications in the areas such as stability theory, chaos and synchronization theory, system norm estimation, and robust control (Liao & Wang, 2003). In this paper, we continue to consider the global dissipativity problem of neural networks, but the activations are not assumed to be continuous.

Synchronization, that means two or more systems share a common dynamical behavior, which can be induced by coupling or by external forcing, is a basis to understand an unknown dynamical system from one or more well-known dynamical systems. From Pecora and Carroll (1990), chaotic synchronization has become a hot topic in nonlinear dynamics due to theoretical significance and potential applications. So far, many types of synchronization have been presented, such as identical or complete synchronization, generalized synchronization, phase synchronization, anticipated and lag synchronization (Liang, Wang, Liu, & Liu, 2008; Luo, 2009). Recently, the quasi-synchronization issue has received a great deal of attention in the literature mainly due to the unavoidability of parameter mismatches between two systems in practical synchronization implementations (Astakhov, Hasler, Kapitaniak, Shabunin, & Anishchenko, 1998; Huang, Li, Yu, &

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Chen, 2009; Jalnine & Kim, 2002; Masoller, 2001; Shahverdiev, Sivaprakasam, & Shore, 2002). Generally, mismatched parameters always implies that the synchronization error could not approach zero with time, but fluctuates. However, it is important to know the region of the synchronization error and control it within a small region around zero, i.e., quasi-synchronization. So, in this paper, the quasi-synchronization of master–response systems with mismatched parameters is investigated.

For these purposed, matrix measure is introduced to deal with matrix inequalities, which can be positive or negative, in comparison to the matrix norm which should always be nonnegative. Owing to these special properties of matrix measure, the results obtained via this approach are usually less restrictive than those via matrix norm (He & Cao, 2009; Vidyasagar, 1993). Another advantage of this approach is avoiding constructing Lyapunov function in the proof. The main contribution of this paper includes three aspects. First, for the differential equations with discontinuous right-hand sides, the concept of Filippov solution (Filippov, 1988) is introduced and the existence of solution is proved for the DNNs. Second, the global dissipativity of Filippov solution is considered by using the matrix measure approach and the generalized Halanay inequalities (Halanay, 1996; Wen, Yu, & Wang, 2008). Third, for the two DNNs with parameter mismatches, the quasi-synchronization issue is discussed also by the matrix measure approach. Furthermore, the complete synchronization between two coupled identical systems can be studied as a special case of the quasi-synchronization.

The rest of the paper is organized as follows. In Section 2, some preliminaries are given. In Section 3, the existence of Filippov solutions of the DNNs is considered and the global dissipativity conditions is obtained by matrix measure approach. In Section 4, the quasi-synchronization of master–response systems with discontinuous activations and parameter mismatches is discussed by the matrix measure method. In Section 5, simulation results aiming at substantiating the theoretical analysis are presented. This paper is concluded in Section 6.

2. Model formulation and preliminaries

In this paper, we consider the following neural networks described by the following differential equations

$$\dot{x}(t) = -D(t)x(t) + A(t)f(x(t)) + B(t)f(x(t-\tau(t))) + J(t), \quad (1)$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ is the state vector associated with the neurons; $D(t) = \mathtt{diag}(d_1(t), d_2(t), \ldots, d_n(t))$ is an $n \times n$ diagonal matrix with $d_i(t) > 0$, $i = 1, 2, \ldots, n$; $A(t) = (a_{ij}(t))_{n\times n}$ and $B(t) = (b_{ij}(t))_{n\times n}$ are the time-varying connection weight matrix and the delayed connection weight matrix, respectively; $f(x) = (f_1(x_1), f_2(x_2), \ldots, f_n(x_n))^T : \mathbb{R}^n \to \mathbb{R}^n$ is a diagonal mapping where f_i, $i = 1, 2, \ldots, n$, represents the neuron input–output activation; $\tau(t)$ is the time-varying delay with $\tau(t) \le \tau$, and $J(t)$ is the external input vector.

Definition 1. Class $\mathcal{F}$ of functions: we call $f(x) \in \mathcal{F}$, if for all $i = 1, 2, \ldots, n$, $f_i(\cdot)$ satisfies: $f_i(\cdot)$ is continuously differentiable, except on a countable set of isolated points $\{\rho_k^i\}$, where the right and left limits $f_i^+(\rho_k^i)$ and $f_i^-(\rho_k^i)$ exist.

In the following, we apply the framework of Filippov in discussing the solution of delayed neural networks (1).

Definition 2 (*Forti & Nistri, 2003*)**.** Suppose $E \subset \mathbb{R}^n$. Then $x \mapsto F(x)$ is called as a set-valued map from $E \hookrightarrow \mathbb{R}^n$, if for each point x of a set $E \subset \mathbb{R}^n$, there corresponds a nonempty set $F(x) \subset \mathbb{R}^n$. A set-valued map F with nonempty values is said to be upper-semi-continuous at $x_0 \in E$ if, for any open set N containing $F(x_0)$, there exists a neighborhood M of x_0 such that $F(M) \subset N$. $F(x)$ is said to have a closed (convex, compact) image if for each $x \in E$, $F(x)$ is closed (convex, compact).

Now we introduce the concept of Filippov solution. Consider the following system

$$\frac{dx}{dt} = f(x), \quad (2)$$

where $f(\cdot)$ is not continuous.

Definition 3 (*Filippov, 1988*)**.** A set-valued map is defined as

$$F(x) = \bigcap_{\delta>0} \bigcap_{\mu(N)=0} K[f(B(x,\delta) \setminus N)], \quad (3)$$

where $K(E)$ is the closure of the convex hull of set E, $B(x,\delta) = \{y : \|y - x\| \le \delta\}$, and $\mu(N)$ is Lebesgue measure of set N. A solution in the sense of Filippov (Filippov, 1988) of the Cauchy problem for Eq. (2) with initial condition $x(0) = x_0$ is an absolutely continuous function $x(t)$, $t \in [0, T]$, which satisfies $x(0) = x_0$ and differential inclusion:

$$\frac{dx}{dt} \in F(x), \quad \text{a.e. } t \in [0, T]. \quad (4)$$

Now we denote

$$\mathbb{F}(x) \triangleq K[f(x)] = (K[f_1(x_1)], \ldots, K[f_n(x_n)]),$$

where $K[f_i(x_i)] = [\min\{f_i(x_i^-), f_i(x_i^+)\}, \max\{f_i(x_i^-), f_i(x_i^+)\}]$, $i = 1, \ldots, n$. We extend the concept of the Filippov solution to the differential equations (1) as follows:

Definition 4 (*Forti & Nistri, 2003*)**.** A function $x : [-\tau, T) \to \mathbb{R}^n$, $T \in (0, +\infty]$, is a solution (in the sense of Filippov) of the discontinuous system (1) on $[-\tau, T)$, if:
(I) x is continuous on $[-\tau, T)$ and absolutely continuous on $[0, T)$;
(II) $x(t)$ satisfies

$$\dot{x}(t) \in -D(t)x(t) + A(t)\mathbb{F}(x) + B(t)\mathbb{F}(x(t-\tau(t))) + J(t), \quad \text{for a.e. } t \in [0, T). \quad (5)$$

Or equivalently,
(II′) there exists a measurable function $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T : [-\tau, T) \to \mathbb{R}^n$, such that $\alpha(t) \in \mathbb{F}(x)$ for a.e. $t \in [-\tau, T)$ and

$$\dot{x}(t) = -D(t)x(t) + A(t)\alpha(t) + B(t)\alpha(t-\tau(t)) + J(t), \quad \text{for a.e. } t \in [0, T) \quad (6)$$

where the single-valued function α is the so-called *measurable selection* of the function $\mathbb{F}$, which approximates $\mathbb{F}$ in some neighborhood of *Graph*$(\mathbb{F})$.

It is obvious that, for all $f \in \mathcal{F}$, the set-valued map $x(t) \hookrightarrow -D(t)x(t) + A(t)\mathbb{F}(x) + B(t)\mathbb{F}(x(t-\tau(t))) + J(t)$ has nonempty compact convex values. Furthermore, it is upper-semi-continuous (Aubin & Cellina, 1984) and hence it is measurable. Here, we remark that all the set-valued functions obtained by Filippov regularization applied to functions $f \in \mathcal{F}$ verify the above several properties. Hence, by the measurable selection theorem (Aubin & Frankowska, 1990), if $x(t)$ is a solution of (1), then there exists a measurable function $\alpha(t) \in K[f(x(t))]$ such that for *a.e.* $t \in [0, +\infty)$, the Eq. (6) is true.

Definition 5 (*Lu & Chen, 2006*)**.** For any continuous function $\theta : [-\tau, 0] \to \mathbb{R}^n$ and any measurable function $\psi : [-\tau, 0] \to \mathbb{R}^n$, such that $\psi(s) \in \mathbb{F}(\theta(s))$ for a.e. $s \in [-\tau, 0]$, an absolute continuous function $x(t) = x(t, \theta, \psi)$ associated with a measurable function $\alpha(t)$ is said to be a solution of the Cauchy problem for