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# Mathematical analysis of the Accordion Grating illusion: A differential geometry approach to introduce the 3D aperture problem

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ABSTRACT

When an observer moves towards a square-wave grating display, a non-rigid distortion of the pattern occurs in which the stripes bulge and expand perpendicularly to their orientation; these effects reverse when the observer moves away. Such distortions present a new problem beyond the classical aperture problem faced by visual motion detectors, one we describe as a 3D aperture problem as it incorporates depth signals. We applied differential geometry to obtain a closed form solution to characterize the fluid distortion of the stripes. Our solution replicates the perceptual distortions and enabled us to design a nulling experiment to distinguish our 3D aperture solution from other candidate mechanisms (see Gori et al. (in this issue)). We suggest that our approach may generalize to other motion illusions visible in 2D displays.

## 1. Introduction

A new visual illusion discovered by one of the authors (S.G.) is reported here. A simple observation shows a perceived deformation from densely juxtaposed parallel lines while moving toward and away from it. This pattern presents two illusory effects: (1) Moving toward and away from the grating creates an expansion/contraction of the stimulus frame perpendicular to the stripes but not parallel with the stripes. One can notice it by paying attention to the stimulus outline in Fig. 1 and observing a horizontal expansion/contraction and less or negligible vertical expansion/contraction. This illusory effect is less vivid than the Rotating Tilted Line Illusion (Gori & Hamburger, 2006) in which a clear rotation emerges. During the observation of the illusion, a loss of the stripes' rigidity (fluid distortion) is observed and the stripes look curved as a function of the observer's motion which is the focus of the present analysis. Foster and Altschuler (2001) reported distortion for a checkerboard grid being viewed by the subject in a back and forth head movement, with the difference being that they used a checkerboard pattern that has many intersections. In the mathematical analysis of our present stimulus, there is no need for line-ends or intersections along the lines to explain the illusion. In fact, our analysis shows that adding more and more intersections systematically decreases the original illusory effect as experimentally shown in Gori et al. (in this issue).

The combination of these phenomenal distortions is similar to the deformations of an accordion while being played, and originated the name of the illusion: the Accordion Grating (AG).

The simple geometrical design of the stimulus renders it a suitable substrate for mathematical analysis (Fig. 1).

This phenomenon is interesting because long lines have no singularities except the line-ends. Having few singularities turned out to be a useful characteristic for the stimuli used in motion analysis (Adelson & Movshon, 1982; Fennema & Thompson, 1979; Grossberg & Mingolla, 1993). Data from single cell electrophysiology shows a delayed true coherent motion signal detection for the mid-portion of long lines, which originates from the motion signals of the line-ends (Pack & Born, 2001).

Without getting into the complexities of binocular vision, we would like to analyze the situation for the monocular condition and see how much of the illusion is predictable based on the geometry of light projection as well as the motion processing units' properties. The result can then be used as a basis to analyze further deviation from it, and uncover other possible factors involved.

## 2. Stimulus configuration

Although the original stimulus design is based on head movement toward and away from the stimulus with a fixed size,

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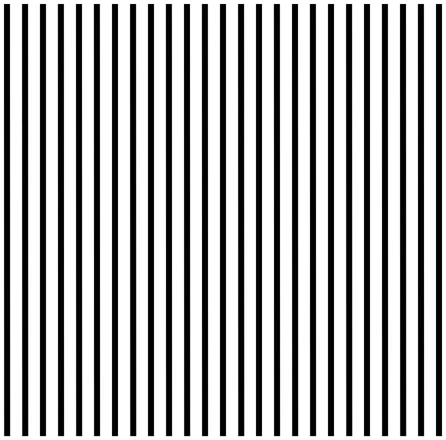

**Fig. 1.** The stimulus of the Accordion Grating. The stimulus is composed of densely juxtaposed parallel lines. Moving toward and away from the stimulus rapidly makes it look as if it centrally bulges and contracts respectively, hence the name, Accordion Grating illusion. The stimulus has a unique design, while its illusory effect is vivid, its geometrical structure is very simple – parallel lines – which makes it a useful candidate for a differential geometry analysis.

in the present study we based our analysis on the fixed position of the head and monitor using, instead, a uniform expansion and contraction of the stimulus. This approach is beneficial, because the presentation is replicable and there is no need to deal with head movement variation across subjects. In fact, a fixed amount of expansion and contraction with a fixed distance from monitor can be replicated in different labs for consistency. Such an approach has been successfully tested in a previous work with Rotating Tilted Line Illusion (RTLI) (Yazdanbakhsh & Gori, 2008) while the very original version of RTLI has a head movement involved (Gori & Hamburger, 2006; Gori & Yazdanbakhsh, 2008). In the few next analysis sections, this fixed distance is represented by $d$ which is constant over time for each condition, but differs for different conditions.

The pattern of grating expansion is shown in Fig. 2. As can be seen, it has a radial pattern as if one prints the stimulus on an elastic sheet and then the sheet expands/contracts radially. Each point of the stimulus is expanded proportional to the distance from the center. As mentioned, this is instead of having the head move toward/away from the stimulus to produce the same radial expansion over the retina.

## 3. Problem formulation

To start approaching the nature of the illusion, we assume the observer's eye nodal point (O) has a distance $OO' = d$ from the stimulus center (O′) where the stimulus is centered at the base of the vertical line of sight (Fig. 3). We index the central bar of stimulus which passes over O′ zero ($n = 0$). The parallel lines to its right are indexed positively ($n = 1, 2, 3 \ldots$) and to its left indexed negatively ($n = -1, -2, -3 \ldots$). The line passing through O′ and orthogonal to parallel lines is called $Ov$ or $x = 0$ line (Fig. 3). Along each of the parallel lines, the distance from the $x = 0$ line is denoted by $x$ (positive on top of $x = 0$ and negative at the bottom). The number of the bars are set to be odd to keep the symmetry ($n = -10$–$10$). For each point of the stimulus A, OA is a projection line.

To evaluate the projection line from each point of the stimulus to the eye's nodal point (O), we consider a point (A) located over the $n$th bar and being offset from the mid perpendicular line by the amount $x$ (Fig. 3). Because the distance between the consecutive parallel lines is $a$, the coordinate of A is therefore $(na, x)$.

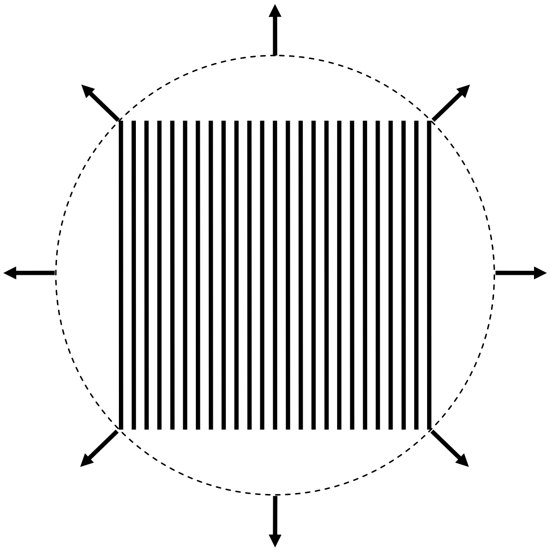

**Fig. 2.** Expanding motion of the AG pattern: each point of the stimulus is expanding proportional to the distance from the center of the expansion.

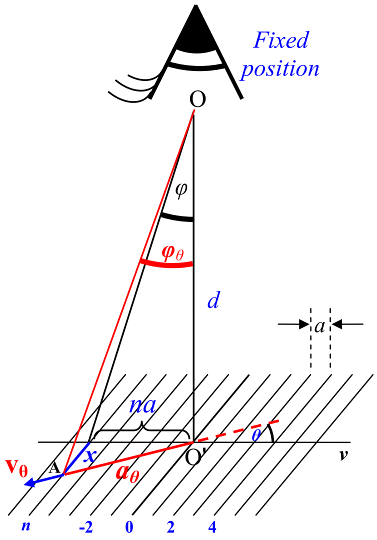

**Fig. 3.** Eye and stimulus position. Fixed position of the eye nodal point (O) and a line of sight perpendicular to the stimulus plane (OO′), in which O′ is set to be the center of the middle line of the stimulus (indexed by $n = 0$). The parallel lines to the right of the $n = 0$ line are indexed by $n = 1, 2, \ldots, 10$ and to the left by $n = -1, -2, \ldots, -10$. The perpendicular line passing through O′ is called the $x = 0$ line or $Ov$. For each point of the stimulus A, OA is a projection line.

### 3.1. Classical aperture problem

If a motion processing unit does not encompass the line-ends or any signal related to them, then the detector is said to have an aperture problem (Gurnsey et al., 2002; Lorenceau et al., 1993; Stumpf, 1911; Wallach, 1935) (Fig. 4(a)). In this case, the motion processing unit can only detect the component of motion perpendicular to the line orientation. Instead, Fig. 4(b) shows the condition in which the line-ends are within the aperture and the true direction of motion can be detected.

### 3.2. Can the classical aperture problem explain the fluid distortion?

Fig. 5 shows the classical aperture problem analysis of the stimulus. Let us again consider an arbitrary point A over the $n$th stimulus line from the center. Due to isotropical expansion, the $n$th line speed or the speed of point A′ (image of A over the line $x = 0$)