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Consensus building in group decision making based on multiplicative consistency with incomplete reciprocal preference relations

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ABSTRACT

In this study, a new method is proposed to address group decision making (GDM) using incomplete reciprocal preference relations (RPRs). More specifically, the multiplicative transitivity property of RPRs is first used to estimate missing values and measure the consistency of preferences provided by experts. Following this, experts are assigned weights by combining consistency weights and trust weights. The former are derived by conducting a multiplicative consistency analysis of the opinions of each expert, whereas the latter are used to measure the degree of trust in an expert harbored by others. Experts with satisfactory consistency and large trust weights should typically be assigned large weights. The consensus level is then checked to determine whether the decision making process moves forward to the selection process. If it is negative, a hybrid method consisting of delegation and feedback mechanisms is used to improve the process of arriving at a consensus. The delegation occurs when some experts decide to leave the process, which is common in GDM involving large numbers of participants. The feedback mechanism, one of the main novelties of the proposed approach, generates different advice for experts based on their consistency and trust weights. Finally, a numerical example was studied to show the practicality and efficiency of the proposed method, and the results indicated that it can provide useful insights into the GDM process.

1. Introduction

Group decision making (GDM) problems are ubiquitous in daily human activities, as people often need to choose from several possible courses of action [25]. Two main processes are needed to achieve a final decision: a consensus process and a selection process. The former is associated with negotiating different opinions to reach a satisfactory level of consensus. Note that a full or unanimous consensus is often not attainable in practice [19]. The latter aims to rank and select an appropriate solution out from a given set of competing alternatives.

Experts usually need to assess multiple alternatives for them to exhibit their preferences based on their understanding of a problem. In recent years, different consensus models have been proposed for GDM with various preference relations. Herrera-Viedma et al. [20] proposed a consensus model for multi-person decision making with different preference relations. Tapia García et al. [16] proposed a consensus model for GDM in which the experts use linguistic interval fuzzy preference relations to represent their preferences. Dong et al. [9] proposed a consensus-based model based on multi-granular, imbalanced two-tuple linguistic preference relations. It works well to manage individual consistency and group consensus while minimizing information loss. Chen et al. [6] presented a method for GDM using group recommendations based on interval fuzzy preference relations and consistency matrices. A hybrid framework considering decision makers' psychological behavior based on prospect theory was proposed for GDM with heterogeneous preference relations [10]. In general, using different preference representation structures often yields different GDM models [12]. However, there are still some new open questions about the use of new preference structures in consensus approaches [4], e.g., to extend the existing models to work with hesitant fuzzy sets, to study consensus models with new preference structures, etc.

However, it is often difficult or even impossible for experts to provide complete and precise assessments of a given problem domain due to its complexity, the pressure to make a decision quickly, or limited expertise, in which cases incomplete fuzzy preference relations are generated. GDM with incomplete comparison matrices has received an increasing amount of research interest, and many methods have been proposed to estimate missing values [32]. For instance, Fan and Zhang [15] proposed a goal programming model for GDM with three formats of incomplete pref-

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erence relations. Gong [17] developed a least-squared method for the priority vector of GDM with incomplete preference relations. Wu et al. [35] investigated trust-based estimation and aggregation methods for GDM with incomplete linguistic information. The concept of relative trust score was defined to estimate unknown preference values to determine the weights assigned to experts. To sum up, three kinds of approaches can be used to deal with incomplete information. The first one involves directly discarding incomplete information [24], the second one penalizes or negatively rates decision makers who provide incomplete information [14], and the third approach features the use of appropriate methodologies to estimate the missing values [31].

Furthermore, consistency is linked to rationality and, therefore, is considered an important factor. For the estimation of missing values, Herrera-Viedma et al. [18] proposed an iterative procedure to compute missing values in incomplete fuzzy preference relations based on additive consistency. Alonso et al. [1] adopted the above procedure and extended it to estimate missing information for different preference formats. Ureña et al. [30] proposed a confidence-consistency driven approach with incomplete reciprocal intuitionistic preference relations for GDM problems. In this approach, confidence level was defined and used to implement both consistency and confidence in the resolution process combined with the consistency level. Wu and Chiclana [33] proposed a novel consensus model based on modeling the multiplicative transitivity property of intuitionistic reciprocal preference relations (RPRs). It takes into account both the consistency index and the proximity index while building consensus, which can lead to higher levels of consistency.

In GDM problems, individual preferences need to be aggregated to obtain a collective one in the selection process. To this end, weights of experts need to be determined. However, in most prevalent studies, weights are given beforehand, which may be unrealistic in practice. Thus, it is essential to provide ways to derive them. Herrera-Viedma et al. [19] recently pointed out that trust relationships between experts can be viewed as a reliable resource for the derivation of the weights of experts. Meanwhile, Wu and Chiclana [34] proposed a trust consensus-based GDM model with interval-valued fuzzy RPRs. The concept of the degree of trust in an expert was developed to determine their importance. Pérez et al. [27] proposed aggregation operators that make full use of information concerning linguistic trustworthiness obtained from experts' social network. Wu et al. [36] proposed a novel social network based GDM model with four-tuple information. In general, the more an expert is trusted, the more the importance assigned to that expert.

Meanwhile, to achieve a satisfactory consensus level, a feedback mechanism [19] is usually incorporated into consensus models to guide the consensus process based on certain consensus measures [12,40,41]. However, in some situations, GDM problems may involve large and even dynamic sets of users. We are at the dawn of a new age of electronic technologies [2], where traditional models do not necessarily meet the requirements and leave some space for new models.

Inspired by above ideals, the authors of this study think that both trust and consistency should be considered throughout the decision making process. Meanwhile, some new procedures should be combined to cope with changing situations involving dynamic sets of users. Hence, this paper proposes a new hybrid approach for GDM based on consistency analysis under incomplete information. The multiplicative transitivity property of RPRs is first used to estimate missing values and measure the level of consistency. Experts are then assigned different degrees of importance based on consistency weights in conjunction with trust weights. The former is defined to measure the consistency of experts' opinions, whereas the latter is used to measure the trust relationship between experts. Some delegation and feedback mechanisms to improve the speed of the process are proposed in this study as well. The delegation process is initiated when some experts leave the decision making process, which in turn influences the trust weights of others. The feedback mechanism helps users change their preferences in the direction of greater consensus based on their consistency weights and trust weights.

The rest of this paper is organized as follows. In the following section, we focus on some preliminary notions used in this paper. The proposed GDM method is detailed in Section 3. In Section 4, a numerical example is studied to illustrate the practicality and feasibility of the proposed method. Some conclusions are discussed in the last section.

2. Preliminaries

To render this paper self-contained, some preliminaries are presented in this section, including some basic definitions associated with fuzzy preference relations.

Fuzzy set theory was introduced by Zadeh [38] to deal with imprecision and uncertainty related to information in a complex environment. Fuzzy set theory treats vague data as probability distributions in terms of set memberships.

Definition 1. (*Reciprocal Preference Relation (RPR)* [7]). An RPR P for set X of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ is characterized by a membership function $\mu_P(x_i, x_j) = p_{ij}$, verifying $p_{ij} + p_{ji} = 1 \forall i, j \in \{1, \ldots, n\}$.

An RPR may be conveniently expressed by matrix $P = (p_{ij})_{n\times n}$, with the following interpretations:

$$P = (p_{ij})_{n\times n} = \begin{bmatrix} 0.5 & p_{12} & \ldots & p_{1n} \\ p_{21} & 0.5 & \ldots & p_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & \ldots & 0.5 \end{bmatrix} \tag{1}$$

where p_{ij} represents the preference value of alternative i over alternative j, and all $p_{ii} = 0.5$. If $p_{ij} = 0.5$, this indicates that there is no difference between the two alternatives; if $p_{ij} > 0.5$, it implies that alternative i is superior to alternative j.

Definition 2. (*Incomplete RPR*). If at least one preference value is unknown for an RPR P, which may occur when an expert does not express a clear attitude toward specific pairs of alternatives, or may be brought about due to time shortages, the preference relation P is called an incomplete RPR.

Definition 3. (*Multiplicative transitivity property of RPR* [33]). An RPR $P = (p_{ij})$ on a finite set of alternatives X is multiplicative transitive if and only if

$$p_{ij} \cdot p_{jk} \cdot p_{ki} = p_{ik} \cdot p_{kj} \cdot p_{ji} \forall i, j, k = \{1, \ldots, n\} \tag{2}$$

is verified by non-zero preference values.

Multiplicative consistency was proposed by Tanino, and is the restriction to the region [0, 1] × [1, 1]\{(0, 1), (1, 0)} of the Cross-ratio uninorm [31]:

$$U(x, y) = \begin{cases} 0, & (x, y) \in \{(0, 1), (1, 0)\} \\ \dfrac{x \cdot y}{x \cdot y + (1 - x)(1 - y)}, & otherwise \end{cases} \tag{3}$$

Eq. (3) can be used to compute consistency-based estimates of the elements of a given RPR. Indeed, any preference value between a pair of alternatives (x_i, x_j), with $i < j$ using another different intermediate alternative x_k, is as follows:

$$mp_{ij}^k = \frac{p_{ik} \cdot p_{kj}}{p_{ik} \cdot p_{kj} + (1 - p_{ik})(1 - p_{kj})} \tag{4}$$

as long as the denominators are not zero.