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A novel quadrature particle filtering based on fuzzy c-means clustering

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ABSTRACT

In this paper, a novel particle filter (PF) which we refer to as the quadrature particle filter (QPF) based on fuzzy c-means clustering is proposed. In the proposed algorithm, a set of quadrature point probability densities are designed to approximate the predicted and posterior probability density functions (pdf) of the quadrature particle filter as a Gaussian. It is different from the Gaussian particle filter that uses the prior distribution as the proposal distribution, the proposal distribution of the QPF is approximated by a set of modified quadrature point probability densities, which can effectively enhance the diversity of samples and improve the performance of the QPF. Moreover, the fuzzy membership degrees provided by a modified version of fuzzy c-means clustering algorithm are used to substitute the weights of the particles, and the quadrature point weights are adaptively estimated based on the weighting exponent and the particle weights. Finally, experiment results show the proposed algorithms have advantages over the conventional methods, namely, the unscented Kalman filter(UKF), quadrature Kalman filter(QKF), particle filter(PF), unscented particle filter(UPF) and Gaussian particle filter(GPF), to solve nonlinear non-Gaussian filtering problems. Especially, to the target tracking in Aperiodic Sparseness Sampling Environment, the performance of the quadrature particle filter is much better than those of other nonlinear filtering approaches.

1. Introduction

The nonlinear filtering problem refers to the state estimation of a nonlinear stochastic system based on noisy observation data obtained from that system over time. This is of paramount importance in many fields of science and engineering, such as navigational and guidance systems, communication, visual tracking and satellite navigation [1,2]. As is well-known, the most widely used filter for the nonlinear filtering problems is the extend Kalman filter (EKF) [2]. However, the performance of the EKF degrades rapidly as the nonlinearity becomes more severe. To alleviate this problem, the Unscented Kalman Filter (UKF) [3] maintains the second order statistics of the target distribution by recursively propagating a set of carefully selected sigma points. This method requires no linearization, and generally yields more robust estimates than the EKF. Ienkaran Arasaratnam and Simon Haykin [4] propose the quadrature Kalman filtering (QKF) by applying the statistical linear regression (SLR) theory to linearize the nonlinear discrete system functions. García-Fernández and Morelande [5] propose a truncated unscented Kalman filtering (TUKF) by approximating the first two moments of the posterior in nonlinear systems with a bijective measurement function. Some interesting relations among some of these KF-type algorithms are given in [6,7]. However, the limitation of these methods is that they do not apply to general non-Gaussian distributions.

To deal with the nonlinear non-Gaussian filtering problem, one popular solution strategy is to use sequential Monte Carlo methods (SMC), also known as particle filters [8,9,10]. The first working particle filter has been reported in [8]. For a recent overview of the field we refer to [11,12]. In particle filter, the choice of the proposal distribution (or the important density) is very important. In general, it is difficult to design such a proposal distribution. Now many proposal distributions have been proposed in the literature. For example, the state transition prior [8], the EKF Gaussian approximation and the UKF proposal [13] are used as the proposal distribution for particle filter. Nevertheless, a major drawback is that the particle filters exhibits a rapid increase in computational complexity as the number of samples increases, and a large part of which comes from resampling. To solve this problem, Jayesh H.K and Petar M.D propose the Gaussian particle filter (GPF) by approximating the posterior mean and covariance of the unknown state variable using importance sampling [14], and the absence of resampling makes it convenient for VLSI implementation and, hence, feasible for practical real-time applications. In [15], they extend the GPF to build Gaussian sum particle filters that can be used for

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models where the predicted and posterior distributions cannot be approximated successfully with a single Gaussian distribution and for models with non-Gaussian noise.

Another way of dealing with nonlinear non-Gaussian problem is the use of soft computing techniques, such as fuzzy logic [16–18]. In [19], Young-Joong et al. propose a fuzzy adaptive particle filter for the localization of a mobile robot, whose basic idea is to generate samples at high-likelihood using a fuzzy logic approach. Shandiz et al. [20] present a particle filtering approach in which particles are weighted using a fuzzy based color model for object. Thomas et al. [21] use an adaptive Gaussian mixture model for background modeling and a fuzzy sequential Monte-Carlo-based tracking algorithm for tracking multiple objects under varying illumination. The use of fuzzy logic to compute the final weight of each particle brings us different benefits compared to the probabilistic approach [22]. First, fuzzy logic approach can approximate the probabilistic distribution in a more flexible way than the probabilistic approach without being restricted to particular aspects of the probability distributions. Secondly, fuzzy logic easily allows us to incrementally add other sources of information by using linguistic variables and rules.

The main contributions of this paper are as follows. First, a novel quadrature particle filter (QPF) based on fuzzy c-means clustering is proposed for nonlinear non-Gaussian filtering problem. Unlike the PF, UPF and GPF, the new QPF uses a set of modified quadrature point probability densities as the proposal distribution, which can effectively enhance the diversity of samples. Second, the distance measure function of fuzzy c-means clustering is defined based on the probability density function, and the probability weights of the particles are substituted by the fuzzy membership degrees provided by a modified fuzzy c-means clustering algorithm, the quadrature point weights are adaptively estimated based on the fuzzy weighting exponent and the particle weights. Finally, the new QPF is analyzed theoretically and studied through two simulation examples.

The rest of the paper is organized as follows. In Section 2, we address the nonlinear Bayesian filtering. In Section 3, we introduce the quadrature particle filtering. Simulation results that compare the performances of all algorithms are presented in Section 4. Finally, some conclusions are provided in Section 5.

2. Nonlinear Bayesian filtering

Consider the nonlinear discrete time dynamic system:

$$x_k = f_k(x_{k-1}, \upsilon_{k-1}) \tag{1}$$

$$z_k = h_k(x_k, e_k) \tag{2}$$

where $f_k : \Re^{n_x} \times \Re^{n_\upsilon} \to \Re^{n_x}$ and $h_k : \Re^{n_x} \times \Re^{n_e} \to \Re^{n_z}$ represent some known nonlinear function, $x_k \in \Re^{n_x}$ is the system state at time k, $z_k \in \Re^{n_z}$ is the measurement vector at time k. $\upsilon_{k-1} \in \Re^{n_\upsilon}$ denotes the process noise, $e_k \in \Re^{n_e}$ denotes the measurement noise.

2.1. Optimal Bayesian filtering

The optimal nonlinear filtering problem is to find the probability density function $p(x_k|z_{1:k-1})$ of the state x_k given the measurement data $z_{1:k}$. The posterior probability density function (PDF) is given by Bayes' formula

$$p(x_k|z_{1:k}) = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})} \tag{3}$$

where $p(z_k|z_{1:k-1})$ is the normalizing constant, $p(z_k|z_{1:k-1}) = \int p(z_k|x_k)p(x_k|z_{1:k-1})dx_k$. The prior PDF $p(x_k|z_{1:k-1})$ of the state at time k can be obtained via the Chapman–Kolmogorov equation,

$$p(x_k|z_{1:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1})dx_{k-1} \tag{4}$$

It is worth noting that the probabilistic model of the state evolution in Eq. (4) has the fact that $p(x_k|x_{k-1}, z_{1:k-1}) = p(x_k|x_{k-1})$ is defined by the process Eq. (1) and the known probability distribution of υ_{k-1}. The likelihood function $p(z_k|x_k)$ can be obtained through the measurement model (2) and the known probability distribution of e_k. In Eq. (3), the measurement z_k is used to modify the prior pdf $p(x_k|z_{1:k-1})$ to obtain the posterior pdf $p(x_k|z_{1:k})$ of the current state x_k.

As well known, when the state and measurement models are linear Gaussian noise, the Kalman filter provides an optimal Bayesian solution. However, for most nonlinear models and non-Gaussian noise problems, the optimal algorithms are impossible to implement, primarily because the pdf updates require integrations that are not practical to implement. As a result, several approximation methods have been proposed, such as particle filtering.

2.2. Particle filtering

In particle filtering, the posterior pdf is approximated by a set of samples with associated weight, which are easily generated from a so-called proposal distribution $\pi(x_k)$. In order to describe the particle filtering algorithm, we suppose that the posterior pdf $p(x_{0:k-1}|z_{0:k-1})$ is approximated by a random measure $\chi_{k-1} = \{x^i_{0:k-1}, w^i_{k-1}\}_{i=1}^{N_s}$, where $\{x^i_{0:k-1}\}_{i=1}^{N_s}$ is a set of support points with associated weights $\{w^i_{k-1}\}_{i=1}^{N_s}$ and N_s is the number of particles [23]. Given the random measure χ_{k-1} and the measurement z_k, the objective is to obtain χ_k based on the random measure χ_{k-1}. Sequential importance sampling methods achieve this by generating particles x^i_k and appending them to $x^i_{0:k-1}$ to form $x^i_{0:k}$, and updating the weights w^i_{k-1} so that χ_k allows for accurate estimates of the unknowns of interest at time k. However, direct sampling from $p(x_{0:k}|z_{0:k})$ is intractable, one can generate particles x^i_k from the proposal distribution $\pi(x)$.

If we use a proposal distribution that can be factored as

$$\pi(x_{0:k}|z_{0:k}) = \pi(x_k|x_{0:k-1}, z_{0:k})\pi(x_{0:k-1}|z_{0:k-1}) \tag{5}$$

and if

$$x^i_{0:k-1} \sim \pi(x_{0:k-1}|z_{0:k-1})$$

and

$$w^i_{0:k-1} \sim \frac{\pi(x^i_{0:k-1}|z_{0:k-1})}{\pi(x^i_{0:k-1}|z_{0:k-1})} \tag{6}$$

We can obtain samples $x^i_{0:k}$ by augmenting each of the existing samples $x^i_{0:k-1}$ with the samples x^i_k that are drawn from $\pi(x_k|x^i_{0:k-1}, z_{0:k})$, then the modified weights can be updated by

$$w^i_k \sim w^i_{k-1}\frac{p(z_k|x^i_k)p(x^i_k|x^i_{k-1})}{\pi(x^i_{0:k-1}x^i_{0:k-1}, z_{0:k})} \tag{7}$$

and the posterior pdf can be approximated as

$$p(x_k|z_{1:k}) \approx \sum_{i=1}^{N_s} w^i_k\delta\left(x_k - x^i_k\right) \tag{8}$$

where $\delta(\cdot)$ is the Dirac delta function. From Eq. (8), it shows that the approximation approaches the true posterior density $p(x_k|z_{1:k})$ as $N_s \to \infty$.

From Eqs. (5) and (7), it is shown that the particle filter consists of recursive propagation of the samples and weights as each measurement is received sequentially. Furthermore, a major problem with particle filtering is the degeneracy phenomenon. In other