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Granule description based on formal concept analysis

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ABSTRACT

Granule description is a fundamental problem in granular computing. Although the spirit of granular computing has been widely adopted in scientific researches, how to classify and describe granules in a concise and apt way is still an open, interesting and important problem. The main objective of our paper is to give a solution to this problem under the framework of granular computing. Firstly, by using stability index, we classify the granules into three categories: atomic granules, basic granules and composite granules. Secondly, in order to improve the conciseness and aptness of granules, we impose additional conditions on minimal generator to define a new term which is called the most apt minimal generator. And then, based on the most apt minimal generator, we put forward methods for the description of atomic granules and basic granules. Moreover, for composite granules, we continue to divide them into three subcategories: $\wedge$-definable granules, $(\wedge, \neg)$-definable granules and $(\wedge, \vee)$-definable granules, and their respective descriptions are provided as well. Finally, some discussions are also made on indefinable granules.

1. Introduction

Granular computing (GrC) is an emerging computing paradigm of information processing, which lies in the scope of cognitive science and cognitive informatics [2,43]. Granular computing studies information and knowledge processing in an abstract way, handles complex information entities in different granules, and allows us to view a phenomenon with different levels of granularity [1,60]. The spirit of granular computing has been adopted frequently in scientific researches, such as philosophy of structured thinking, structured problem solving, and structured information processing. In this sense, all the methods which treat information in this perspective will fall into the scope of granular computing [12,26,34,37].

To put it simply, information granules are collections of entities which are arranged together due to their similarity, functional or physical adjacency, coherency, and so on [29,53,69]. At present, granular computing is not a coherent set of methods or principles but rather a theoretical perspective, which encourages researchers to deal with knowledge at different levels of abstraction or generalization [9,40,52,66,68]. It often granulates the universe of discourse into a family of disjoint or overlapping granules. Based on this idea, different views of the universe of discourse can be linked together, and a hierarchy of granulations can be established. Thus, one of the main directions in the study of granular computing is to deal with the construction, interpretation, and representation of granules [50].

Rough set theory (RST), as an efficient tool of granular computing, presented by Pawlak [31], has drawn many attentions from researchers over the past thirty-four years [14,19,33,48,54,67,70,71]. As is well known, the original idea of rough set theory is to partition the universe of discourse into disjoint subsets by a given equivalence relation, and then by using the obtained disjoint subsets, target sets are characterized by means of the so-called lower and upper approximations.

Rough sets were used to describe a target set by the lower and upper approximations under one granulation, but multiple granulations are sometimes required to approximate a target set when dealing with multi-scale or multi-source data sets [35,36,51]. Under such a circumstance, pessimistic multigranulation rough sets and optimistic multigranulation rough sets were proposed for applying multi-source information fusion. These information fusion strategies were soon extended to cater the cases such as incomplete, neighborhood, covering and fuzzy environments [13,24,25,39,55,59]. Moreover, a byproduct is that "AND" and "OR" decision rules can be derived from decision systems with the pessimistic and optimistic multigranulation rough sets [35,36], which was further exploited by Yang et al. [58] and Li et al. [23] in terms of local and global measurements of the "AND" and "OR" decision rules.

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Rough set theory is actually related to and complementary with *formal concept analysis* (FCA) [17,27,30,49], and more and more attention [5,15,17,18,38,46,62] has been paid to comparing and combining rough set theory and formal concept analysis. Object-oriented concept lattice was introduced in [63] by incorporating lower and upper approximation ideas into concept-forming operators, and it was further elaborated in [28,44]. In the meanwhile, some studies were also conducted by integrating formal concept analysis with granular computing, such as granular rule acquisition [20,21,50,56], concept learning [22], fuzzy information granule description [57], granular reduct [50], granule transformation and irreducible element judgment [47]. In addition, rough set theory has been related to granular computing [35,36,51,61], and vice versa.

However, as one of the most important tasks in granular computing, granule description has attracted little attention. This problem deserves to be investigated since it can not only help us to have a better understanding and comprehension of granules, but also shed some light on the unsolved problem "why some concepts are psychologically simple and easy to learn, while others seem difficult" [3]. Motivated by this problem, the main objective of this paper is to propose a granular description method based on formal concept analysis. In formal concept analysis, there are two types of granules that make sense. One is the granules formed by the extents of formal concepts, and the other is the ones formed by individual objects. Some studies have shown that the latter plays a very important role and has a strong correlation with object concepts [7,47], object granules [50] and granular concepts [22]. In fact, besides these two types of granules, there still exist other types of granules. How to classify granules and what is the classification criteria are still open problems. We will give solutions to these two problems in Section 3.

Moreover, how to describe any granule in a concise and apt way is a fundamental issue in solving granule description problem under the framework of granular computing. As mentioned above, there are two types of granules that make sense and have attracted many researchers' attentions. In fact, the descriptions of the first type of granules can be realized by using minimal generators [8]. However, it is worth noticing that for a given concept, there may be more than one minimal generator, and the most apt and concise one is the best choice. Therefore, we need additional constraints to refine the results, and this issue will be investigated in Section 4.

For the description of individual object, although it gives arise to the problem of finding a reduct of the original description of the object, it differs from the description of the first type of granules, since at the most time it is impossible to distinguish one object from the other by the attributes it possesses. Thus, it is necessary to point out which attributes it does not possess. This issue will be considered in Section 5.

In Section 6, we will put forward a method for the description of other types of granules. In Section 7, we firstly discuss the problem of describing indefinable granules, and then we make a comment on the connection of RST and FCA from the viewpoint of granule description. Some discussions and remarks are given in Section 8.

2. Related theoretical foundations

In this section, the involved notions are introduced briefly. In the introduction of logic language L, we make some necessary modifications to the definition of $m(\varphi)$ (i.e., the meaning of formula φ) in order to get a finer semantic meaning.

2.1. Logic language L

Logic language L, which adopts and modifies the decision logic language used in rough set theory, enables to formally represent and interpret rules in the process of knowledge discovery [32,64]. In order to obtain stronger description ability, the logic language L is built on a set of atomic formulas.

Atomic formulas, which are denoted by $A = \{p, q, \dots\}$, provide a foundation for complex knowledge representation. By using logic connectives such as $\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$, compound formulas can be built recursively. If φ and ψ are formulas, then so are $\neg\varphi$, $\varphi\wedge\psi$, $\varphi\vee\psi$, $\varphi \rightarrow \psi$ and $\varphi\leftrightarrow\psi$.

In mathematical logic, a literal is an atom or its negation. Moreover, literals can be divided into two types: a positive literal is just an atom; a negative literal is the negation of an atom.

For a given formula φ, let $lit(\varphi)$ denote both the positive and negative literals contained in φ. Moreover, let $|lit(\varphi)|$ denote the cardinality of $lit(\varphi)$, i.e., the number of literals contained in $lit(\varphi)$.

For example, let $\varphi = g\wedge\neg h$. Then we have $lit(\varphi) = \{g, \neg h\}$ and $|lit(\varphi)| = 2$.

The semantics of the language L is defined as a pair $M = (D, K)$, where D is a nonempty set of individuals and K is available knowledge about individuals of D.

Let p be an atomic formula and x an individual. By using knowledge K, if x satisfies p, then we have the denotation as $x\mapsto p$.

The meaning of the formula φ is the set of individuals which satisfy this formula, and is defined by the following equation:

$$m(\varphi) = \{x | x \in D, x \mapsto \varphi\}.$$

Considering the above equation in the reverse direction, we define the description of subset A as $m^{-1}(A)$, where

$$m^{-1}(A) = \varphi \text{ such that } m(\varphi) = A.$$

For a given granule A, in order to get the most concise and apt description, we define a new function $d(A)$ instead of using the function $m^{-1}(A)$.

Definition 1. Let A be a granule. The description of A is defined by $d(A)$, where

$$d(A) = \varphi \text{ such that } m(\varphi) = A \text{ and for any formula } \psi\text{, we have}$$

(i) if $m(\psi) = A$, then $|lit(\psi)| \geq |lit(\varphi)|$;
(ii) if $m(\psi) = A$ and $|lit(\psi)| = |lit(\varphi)|$, then $m(\wedge p)_{p\in lit(\varphi)-lit(\psi)} \subseteq m(\wedge q)_{q\in lit(\psi)-lit(\varphi)}$.

Condition (i) ensures the conciseness of the description. That is, the description contains the fewest literals. Condition (ii) ensures the aptness of the description. That is, the literals contained in the description have the smallest extent.

2.2. Overview of granular computing and basic notions on FCA

Granular computing aims to represent and solve complicated problems in the procedure of granularity transformation [1]. Internal structure of a granule, collective structure of a family of granules and hierarchical structure of a web of granules are the three most important parts of a granular structure [65].

Given a domain D, all possible granules form a power set of D, denoted as 2^D. Here, the part we are interested in is only subsystem of 2^D. For example, in FCA [7], the granules which deserve our attention are the extensions of formal concepts, while in knowledge spaces theory [41], the ones turn to be the feasible knowledge statements.

FCA is generally an appropriate framework for building categories which are defined as object sets sharing some attributes, irrespectively of a particular domain of application.

Given a formal context $K = (G, M, I)$, where G is called a set of objects, M is called a set of attributes, and the binary relation $I\subseteq G \times M$ specifies which objects have what attributes. Moreover, the derivation functions $f(\cdot)$ and $g(\cdot)$ are defined for $A\subseteq G$ and $B\subseteq M$ as