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A new method for group decision making with incomplete fuzzy preference relations

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ABSTRACT

This paper presents a new method to coping with group decision making with incomplete fuzzy preference information. To do this, it first defines an additively consistent index of fuzzy preference relations, and then gives a method to calculating the priority vector for additively consistent fuzzy preference relations. When the individual fuzzy preference relation is incomplete, a goal programming model is constructed, by which the missing values can be obtained. Then, an iterative approach to obtain the acceptably additive consistency of fuzzy preference relations is introduced. After that, an induced hybrid weighted aggregation (IHWA) operator is presented to aggregate the collective fuzzy preference relation. The main features of this aggregation operator are that the group consistency is no smaller than the highest individual inconsistency, and the group consensus is no smaller than the smallest consensus between the individual fuzzy preference relations. As a series of development, an algorithm based on the acceptable consistency and the group consensus is developed. Finally, three examples are given to show the efficiency and feasibility of the developed procedure, and comparisons are also offered.

1. Introduction

In the processes of multi-criteria decision making (MCDM) using Analytic Hierarchy Process (AHP) [57], it usually needs the experts to give their preference information. According to the given preference values, there are three types of preference relations: the reciprocal preference relations [57], the fuzzy preference relations [51,59] and the linguistic preference relations [29]. No matter which kind of preference relations is used, the main issue is how to derive the priority vector. There are usually two kinds of methods. One of which is based on consistency analysis [5,14,18,42,43,53,58,61,69,74,80]; the other adopts the programming models [3,4,6,38,44,45,50,54,56,62,63,82,83]. The main disadvantage of the latter is their failure to handle inconsistency, which may lead to a misleading solution [43].

Since the decision-making problems become more and more complex, it is difficult or even impossible for a single expert to consider all aspects of a problem [37]. In this case, it usually requires more than one expert to make decisions for one problem, which is the so-called group decision making [33,34,49]. To obtain the global priority vector, there are usually two different approaches: the aggregation of individual judgements (AIJ) and the aggregation of individual priorities (AIP) [22]. For the former, using the consistent index introduced by Tanino [59], Xu [73] proved that the weighted geometric mean complex judgment matrix (WGMCJM) is of acceptable consistency under the condition that each individual judgment matrix is of acceptable consistency. With respect to the geometric consistency index (GCI) [1], Escobar et al. [19] showed that the inconsistency of the group is smaller than the largest individual inconsistency for both aggregation approaches (AIJ and AIP) by the weighted geometric mean method (WGMM) as the aggregation procedure and the row geometric mean method (RGMM) as the prioritization procedure. Chiclana et al. [8] presented a framework for integrating individual consistency into consensus model, which is composed by two processes: individual consistency control process and consensus reaching process. Dong et al. [17] presented two consensus indices: the geometric cardinal consensus index (GCCI) and the geometric ordinal consensus index (GOCI). Then, the authors introduced two algorithms to obtain the acceptably consistent collective reciprocal judgment matrix. Based on the individual consistency level (CL) [26], Zhang et al. [84] developed three models to adjust the consistency of individual fuzzy preference relations. The programming methods for group decision making can be seen in the literature [23,64,70,75,86],

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and group decision making with linguistic preference relations are considered in [25,48,76,77].

All above-mentioned researches are based on the complete preference information. However, in some situations, because of various reasons, such as time pressure, lack of knowledge or data, and their limited expertise related to the problem domain, the experts may only provide incomplete preference relations, namely, preference relations with some values unknown [20,35,36,78]. Xu [78] researched the incomplete fuzzy preference relations using two goal programming models, and Gong [24] developed a least-square method to the priority vector of group decision making with incomplete fuzzy preference relations. These two methods give the ranking of alternatives using the known fuzzy preference relations. However, neither of them considers the individual consistency and the group consensus. Different to the methods given in [71,75], Herrera-Viedma et al. [26] proposed an iterative procedure based on the additive consistency [59] to estimate the missing values of an incomplete fuzzy preference relation, which only uses the preference values provided by the same expert. Then, the authors introduced a selection process based on the fuzzy majority and the additive consistency induced ordered weighted averaging (AC-IOWA) operator. After the pioneer work of Herrera-Viedma et al. [26], Herrera-Viedma et al. [32] further researched group decision making with incomplete fuzzy preference relations using the iterative procedure given in [26], which considers group consensus and individual consistency; Alonso et al. [2] adopted the iterative procedure in [26] to present a procedure to estimate the missing information for different incomplete preference formats, such as fuzzy, multiplicative, interval-valued, and linguistic preference relations. Inspired by Herrera-Viedma et al. [26], Lee [41] presented a method for group decision making with incomplete fuzzy preference relations based on the additive consistency and the order consistency. Later, Chen et al. [7] pointed out that there are some drawbacks of Lee's method and presented an improved method. Porcel and Herrera-Viedma [52] developed an approach to recommender system to university digital libraries, which allows the users to provide their preferences by means of incomplete fuzzy linguistic preference relation. To avoid loss of information, the authors adopted 2-tuple linguistic computational model [31]. More researches about the application of incomplete fuzzy linguistic preference relation in recommender systems can be seen in the literature [28,47,55]. Furthermore, Xu [72] studied group decision making with four formats of incomplete preference relations. To obtain the final priority vector, the author first constructed the associated optimization models to convert the different preference formats into fuzzy preference relations, and then derived the priority vector by solving the established optimization programming model. The main issue of this method is that it neither considers consensus nor analyzes consistency. Based on the facts that many approaches to integrating different preference representation formats have been proposed [9–11,15,21,27,30,85], and there are many reasons to choose the fuzzy preference relations as the integrating element [26]. In this paper, we continue to research group decision-making problems with incomplete fuzzy preference relations. There are main five advantages of the new method: (i) it is based on the individual consistency and the group consensus analyses; (ii) when the individual fuzzy preference relation is incomplete, by the building goal programming model it can cope with more general cases and get more reasonable missing values than some existing methods; (iii) it calculates the collective fuzzy preference relation by considering the importance of the experts and the ordered positions; (iv) the collective fuzzy preference relation derived by this method has the bigger consistent degree than the smallest individual consistency; (v) the group consensus is bigger than the smallest consensus between individuals.

This paper is organized as follows: Section 2 briefly reviews some basic concepts related to fuzzy preference relations. Then, it analyzes the principles of some existing approaches and points out some their issues. Section 3 defines a consistent index of fuzzy preference relations and gives a method to derive the priority vector. To cope with the unacceptably consistent case, we introduce an iterative method to improve the consistent level, by which the limit of the additive consistency of any fuzzy preference relation is equal to 1. Meanwhile, to cope with the situation where the fuzzy preference information is incomplete, the goal programming model is constructed, by which the missing values can be obtained. It is worth pointing out that this method can cope with more situations and more reasonable than some existing ones. Section 4 discusses the group consensus problem and defines a group consensus index. Then, it introduces an induced hybrid aggregation operator to calculate the collective fuzzy preference relation. This operator applies the additively consistent levels of individual fuzzy preference relations as the order-inducing variables, which endows the higher consistent individual fuzzy preference relation with more importance. Then, an algorithm for group decision making with incomplete fuzzy preference information is developed, which does not only consider the individual consistency but also indicate the group consensus. Section 5 applies three examples to show the application of the new method, and the associated comparisons are also offered. The conclusion is made in the last section.

2. Preliminary

Among the different preference formats in decision making using analytic hierarchy process (AHP), fuzzy preference relations are one of the most common used preference formats for their efficiency and simplicity. Throughout the paper, let $X = \{x_1, x_1, \ldots, x_n\}$ be the set of compared objects including alternatives, criteria, experts. To consider the preference relation of objects, Orlovsky [51] introduced us to a fuzzy preference relation on X as follows:

Definition 1 [51]. A fuzzy preference relation R on a set of objects X is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function μ_R: $X \times X \to [0,1]$.

According to Definition 1, a fuzzy preference relation R on X can be conveniently expressed by an $n \times n$ matrix $R = (r_{ij})_{n\times n}$, where $r_{ij} = \mu_R(x_i,x_j)(i,j = 1, 2, \ldots, n)$ is interpreted as the preference degree or intensity of the alternative x_i over x_j. When $r_{ij} = 0.5$, it indicates indifference between x_i and x_j ($x_i \sim x_j$); $r_{ij} > 0.5$ indicates that x_i is preferred to x_j ($x_i \succ x_j$). In general, $R = (r_{ij})_{n\times n}$ satisfies the additive reciprocity property, namely, $r_{ij} + r_{ji} = 1$ for all i, $j = 1, 2, \ldots, n$. Without loss of generality, in this paper we always assume that R is additive reciprocal. Consider the consistency a fuzzy preference relation $R = (r_{ij})_{n\times n}$, Tanino [59] introduced the following concept of additive consistency.

Definition 2 [59]. The fuzzy preference relation $R = (r_{ij})_{n\times n}$ is additively consistent, if it satisfies

$$r_{ij} = r_{ik} + r_{kj} - 0.5, \tag{1}$$

for all $i, k, j = 1, 2, \ldots, n$ with $i < k < j$ and $r_{ij} + r_{ji} = 1$.

Herrera-Viedma et al. [26] classified three cases of Eq. (1), which are respectively denoted by

(i) $r_{ij}^{k,1} = r_{ik} + r_{kj} - 0.5$ by $r_{ij} = r_{ik} + r_{kj} - 0.5$;
(ii) $r_{ij}^{k,2} = r_{kj} - r_{ki} + 0.5$ by $r_{kj} = r_{ki} + r_{ij} - 0.5$;
(iii) $r_{ij}^{k,3} = r_{ik} - r_{jk} + 0.5$ by $r_{ik} = r_{ij} + r_{jk} - 0.5$.