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Identification of uncertain nonlinear systems: Constructing belief rule-based models ☆

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The objective of this paper is to construct reliable belief rule-based (BRB) models for the identification of uncertain nonlinear systems. The BRB methodology is developed from the evidential reasoning (ER) approach and traditional IF–THEN rule based system. It can be used to model complicated nonlinear causal relationships between antecedent attributes and consequents under different types of uncertainty. In a BRB model, various types of information and knowledge with uncertainties can be represented using belief structures, and a belief rule is designed with belief degrees embedded in its possible consequents. In this paper, we first introduce the BRB methodology for modelling uncertain nonlinear systems. Then we present a comparative analysis of three BRB identification models through combining the BRB methodology with nonlinear optimisation techniques. The novel BRB identification models using l_∞-norm and minimising mean uncertainties in belief rules (MUBR) show remarkable capabilities of capturing the lower and upper bounds of the interval outputs of uncertain nonlinear systems simultaneously. Trade-off analysis between identification accuracy and interval credibility are briefly discussed. Finally, a numerical study of a simplified car dynamics is conducted to demonstrate the capability and effectiveness of the BRB identification models for the modelling and identification of uncertain nonlinear systems.

1. Introduction

Identification of an uncertain nonlinear system is mainly concerned with characterising the unknown nonlinear system on the basis of measured input–output data in an uncertain environment [26]. It is of fundamental importance in predictive control, fault diagnosis, signal processing and decision analysis, since most real-life systems are nonlinear, and are often associated with uncertainties due to noises, unpredictable disturbance and measurement errors, uncertain physical parameters, incomplete knowledge, etc. [1,43,40,30,20]. Thus a great difficulty in applying traditional identification techniques is dealing with those uncertainties [2,4]. Over past few decades, extensive studies have been conducted for effectively identifying uncertain nonlinear systems, especially with the advent of neural network and fuzzy rule-based system techniques [34,29,17,12]. Neural networks and fuzzy rule-based systems often outperform traditional identification methods in terms of both approximation accuracy and identification reliability. Sjöberg et al. [34] provided a unified overview on nonlinear black-box system identification models with structures based on neural networks and fuzzy rules. Tseng and Chen [40] applied the Takagi–Sugeno fuzzy model to model uncertain nonlinear systems and proposed H_∞ fuzzy filter design for state estimation of nonlinear discrete systems with bounded but unknown disturbance. Nelles [29] provided an in-depth analysis of nonlinear system identification methods, including linear and polynomial approximation, neural networks and fuzzy models. Zheng et al. [49] presented a robust Takagi–Sugeno fuzzy control model for nonlinear systems with both parameter uncertainty and external disturbance. Choi [12] developed an adaptive fuzzy control system for uncertain Takagi–Sugeno fuzzy models with norm-bounded uncertainty on the basis of the variable structure control (VSC) theory. González-Olvera and Tang [17] presented a continuous-time recurrent neuro-fuzzy network for the black-box identification of a class of dynamic nonlinear systems.

However, these aforementioned methods may not be directly applicable to some real-world uncertain nonlinear systems, because they involve some restrictive assumptions, such as Gaussian-distributed noises, deterministic disturbances and

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bounded uncertain parameters [5]. In addition, the outputs of uncertain systems are usually represented on the basis of predicted point values. In uncertain systems, any uncertainties usually generate uncertain outputs and the commonly used point prediction delivers no information about different kinds of uncertainties [28]. Therefore, the identification of using prediction intervals (i.e., estimated range of predictions) to model uncertain outputs has attracted much attention in recent years. Shrestha and Solomatine [33] employed prediction intervals to identify the uncertainty of the model outputs using different machine learning techniques, such as locally weighted regression and artificial neural network. Mazloumi et al. [28] discussed how uncertainty arising from both model structure and training data can be quantified through constructing prediction intervals in neural networks. Khosravi et al. [21] proposed a lower upper bound estimation (LUBE) method for construction of neural network-based prediction intervals. The LUBE method uses a neural network with two outputs to construct upper and lower bounds of the prediction intervals, and both interval width and coverage probability are incorporated into the training objective function. However, due to the black-box nature of neural networks, these neural network-based prediction intervals cannot be interpreted in an explicit way and does not reflect uncertain knowledge. Furthermore, on the basis of fuzzy identification methodology, Škrjanc et al. [35,36] presented an interval fuzzy model (INFUMO) to model a class of nonlinear systems with interval parameters. It results in a lower and upper fuzzy model or so called a fuzzy model with lower and upper parameters. Linear programming techniques are used to find the set of lower and upper parameters using the l_∞-norm as the optimality criterion. The idea behind the INFUMO is to find optimal lower and upper bound fuzzy systems that define a prediction interval which encloses all the measured input–output data [37]. However, in INFUMO the lower and upper bound fuzzy models are independent from each other, and it may incur improper identification results with invalid lower and upper bounds if there is no enough training data available in some regions of the input space [36,31].

The motivation of this paper is to construct reliable belief rule-based (BRB) models for the identification of uncertain nonlinear systems. The BRB methodology is developed on the basis of the Dempster-Shafer theory of evidence [14,32], decision theory [47,16], traditional IF–THEN rule-based systems [18,13,24] and relevant artificial intelligence (AI) techniques [39,22,15]. The BRB methodology has an inherent capability of dealing with various types of uncertainty. In a BRB system, various types of information and knowledge with uncertainties can be represented using belief structures, and a belief rule is designed with belief degrees embedded in its possible consequents. The belief structure used in both belief rules and inference processes provides a unified scheme to model uncertain system outputs caused by vagueness, fuzziness, or incompleteness, etc. A belief distribution with incompleteness can be transformed to a prediction interval in a straightforward way. Compared with traditional rule based systems, BRB systems provide a more informative knowledge representation scheme for both quantitative data and qualitative information with uncertainties, and it is also capable of approximating complicated nonlinear causal relationships between antecedent inputs and output [45]. In recent years, it has been successfully applied in various areas, such as fault diagnosis, system identification, forecasting and decision analysis [45,46,42,3,50,6,48,25].

The rest of the paper is organised as follows. In Section 2, the BRB methodology for modelling uncertain nonlinear systems is introduced. In Section 3, a comparative analysis of three identification models for uncertain nonlinear systems is presented through combining the BRB methodology with nonlinear optimisation techniques. The novel BRB interval identification model using l_∞-norm and minimising mean uncertainties in belief rules are capable of capturing the lower and upper bounds of the interval outputs of uncertain nonlinear systems simultaneously. Numerical studies and trade-off analysis are introduced to illustrate the performance of the BRB identification models. In Section 4, a numerical study is conducted to demonstrate the superior capability of the proposed BRB identification models for the identification of uncertain nonlinear systems. The paper is concluded in Section 5.

2. Modelling uncertain nonlinear systems with belief rules

The modelling and identification of an uncertain nonlinear system is basically to characterise an unknown relationship between a set of input variables $\boldsymbol{x} = \{x_i;\ i = 1, \ldots, M\}$ and a dependent variable y using a finite number of input–output datasets $\{\boldsymbol{x}_t;y_t\}$, $t = 1, \ldots, T$, where y_t denotes the measured output at the sampling time t. Correspondingly a BRB identification model can be described as the set of input variables and a vector of parameters which are combined in a nonlinear manner to predict the behaviour of the dependant variable. The model can be represented by

$$y_t = f(\boldsymbol{x}_t; \boldsymbol{P}^*) + \varepsilon_t; \quad t = 1, \ldots, T \tag{1}$$

where $\boldsymbol{P}^*$ is used to denote the optimal values of the set of parameters, and ε_t is the model error which is usually assumed to be independently and normally distributed. The analytical model $f(\boldsymbol{x};\boldsymbol{P}^*)$ is decided by model structure, belief rules and inference process which will be introduced in the following sections.

2.1. Belief rules

To model the uncertain nonlinear relationship between the set of input variables $\boldsymbol{x}$ and the dependent variable y, a belief rule base which is made up of a finite number of belief rules can be constructed. Typically, a belief rule is given in the following form [45,6].

$$R_k: \begin{array}{l} \text{IF } x_1 \text{ is } A_1^k \wedge x_2 \text{ is } A_2^k \wedge \cdots \wedge x_{M_k} \text{ is } A_{M_k}^k \\ \text{THEN}\{(D_1, \beta_{1,k}), (D_2, \beta_{2,k}), \ldots, (D_N, \beta_{N,k})\}, \left(\sum_{n=1}^{N} \beta_{n,k} \leqslant 1\right) \\ \text{with rule weight } \theta_k \\ \text{and weight of variables } \delta_{1,k}, \delta_{2,k}, \ldots, \delta_{M_k,k}, k \in \{1, \ldots, K\}, \end{array} \tag{2}$$

where $x_1, x_2, \ldots, x_{M_k}$ denote the antecedent variable in the kth rule, and these variables belong to the complete set of input variables $\boldsymbol{x} = \{x_i; i = 1, \ldots, M\}$. $A_i^k (i = 1, \ldots, M_k)$ is the referential value taken by the ith antecedent variable in the kth rule and $A_i^k \in \boldsymbol{A}_i$. $\boldsymbol{A}_i = \{A_{i,j};\ j = 1, \ldots, J_i\}$ denotes the set of referential values for the ith antecedent variable and J_i is the number of the referential values. As a set of referential values needs to be defined for each antecedent variable, and the antecedents in a belief rule is a combination of the referential values of antecedent variables, a BRB model essentially decomposes the input space of an uncertain nonlinear system into multiple subspaces. The number of referential values on each antecedent variable decides the granularity and interpretability of the subspaces [7]. $\beta_{n,k}(n = 1, \ldots, N;\ k = 1, \ldots, K)$ represents the belief degree to which the consequent element D_n is believed to be the consequent, given the logical relationship of the kth rule IF x_1 is $A_1^k \wedge x_2$ is $A_2^k \wedge \cdots \wedge x_{M_k}$ is $A_{M_k}^k$. The element D_n in the set of consequent elements $\boldsymbol{D} = \{D_n;\ n = 1, \ldots, N\}$ can either be a conclusion or an action and a subset of elements can also be part of the consequent [42]. The nonlinear inference process of BRB models which will be discussed below is based on the belief distribution $\{(D_n,\beta_{n,k});\ n = 1, \ldots, N\}$. If $\sum_{n=1}^{N} \beta_{n,k} = 1$, the kth rule is said to be complete; otherwise, it is incomplete, and the incomplete belief degree