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Knowledge reduction in formal fuzzy contexts

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ABSTRACT

Knowledge reduction is a basic issue in knowledge representation and data mining. Although various methods have been developed to reduce the size of classical formal contexts, the reduction of formal fuzzy contexts based on fuzzy lattices remains a difficult problem owing to its complicated derivation operators. To address this problem, we propose a general method of knowledge reduction by reducing attributes and objects in formal fuzzy contexts based on the variable threshold concept lattices. Employing the proposed approaches, we remove attributes and objects which are non-essential to the structure of a variable threshold concept lattice, i.e., with a given threshold level, the concept lattice constructed from a reduced formal context is made identical to that constructed from the original formal context. Discernibility matrices and Boolean functions are, respectively, employed to compute the attribute reducts and object reducts of the formal fuzzy contexts, by which all the attribute reducts and object reducts of the formal fuzzy contexts are determined without changing the structure of the lattice.

1. Introduction

The theory of formal concept analysis (FCA), proposed by Wille [11,32], stimulates mathematical approach to conceptual data analysis and knowledge processing. The main concern of FCA is the lattice structure, called a concept lattice, induced by a binary relation between a pair of sets constructed by the formal concepts embedded in a concept hierarchy. A formal concept is an (objects, attributes) pair, the set of objects is referred to as the extent of the concept consisting of all objects belong to the concept, and the set of attributes is called the intent of the concept that include all attributes shared by the objects. In the last two decades, FCA has been successfully applied to information retrieval, knowledge discovery, data mining, machine learning, software engineering, etc. [5,6,8,10,15,16,18,21,22,24,30,39].

FCA is formulated on the basis of a formal context, which is a binary relation between a set of objects and a set of attributes. Numerous theoretical investigations and practical applications of FCA have been made over the years. Extensions of FCA theory via fuzzy logic reasoning or fuzzy set theory also have been attempted in recent years. In real-life problems, formal fuzzy contexts are more common than their crisp counterparts. Binary fuzzy relations have been used to analyze relations between objects and attributes, and several generalizations to formal fuzzy concept have been made. For instance, Belohlavek [1–3] proposed fuzzy concepts in formal fuzzy contexts based on a residuated lattice, and the proposed fuzzy concept lattice hold almost all properties that in a classical concept lattice. Georgescu and Popescu [12,27] generalized Belohlavek's definition of fuzzy concepts and discussed a general approach to fuzzy FCA. Medina and Ojeda-Aciego [25] developed so-called t-concept lattice as a set of triples associated to graded tabular information interpreted in a non-commutative fuzzy logic. Jaoua and Elloumi [14] extended the binary fuzzy relation to a real-set binary relation and introduced the corresponding Galois lattices. On the other hand, Krajči [16], Yahia and Jaoua [36] independently proposed a "one-sided fuzzy concept", one advantage of their definitions is that the number of generated formal concepts is greatly reduced. Zhang et al. [40] further constructed the "variable threshold concept lattices", i.e. crisp-fuzzy variable threshold concepts and fuzzy-crisp variable threshold concepts, in which the "one-sided fuzzy concept" becomes a special case (threshold being equal to 1).

One of the main directions in the study of FCA is knowledge reduction achieved by reducing attributes and objects in formal contexts while keeping the lattice structures unchanged. Knowledge reduction is an important issue in knowledge representation and data mining [7,13,19,28,34,37,38]. Many types of knowledge reduction along the line of reducing attributes have been proposed for the analysis of information tables and decision tables. Since a formal context can be regarded as an information table, it is natural

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to consider knowledge reduction in FCA. In formal contexts, knowledge reduction is the search for a subset of attributes which can provide the same structure of the original concept lattice. In recent years, there have been a rapid growth of interests in knowledge reduction in classical formal contexts (object set and attribute set are crisp sets) [9,11,17,20,23,26,29,31,33,35,41].

As we know, the number of formal concepts in a formal fuzzy context increases dramatically than that in a classical formal context. Hence, the structure of a concept lattice derived from a formal fuzzy context is more complicated than the one in a classical formal context. Thus the study of knowledge reduction in a formal fuzzy context becomes even more meaningful. By reduction, the concepts in a reduced fuzzy context have a simpler form, making the lattice a better representation of knowledge.

Unfortunately, the methods of knowledge reduction for classical formal contexts are not applicable in the reduction of formal fuzzy contexts. The issues of knowledge reduction in formal fuzzy contexts has recently attracted attention in the research community. Belohlavek et al. [4], for instance, proposed a method to reduce the number of formal fuzzy concepts by only keeping the so-called crisply generated fuzzy concepts which are generated from some crisp subset of attributes, leaving out non-crisply generated fuzzy concepts. In Belohlavek et al. reduction method [4], the number of fuzzy concepts and the hierarchy of the lattice are changed, resulting in a possible loss of knowledge. Based on the Lukasiewicz implication, Elloumi et al. [9] gave a multi-level conceptual data reduction approach via the reduction of the object sets by keeping the minimal rows in a formal context. However, we have noticed that some of the attributes are still redundant in the row reduced formal context, and Elloumi et al. did not consider eliminating the reducible attributes. Furthermore, based on the multi-level formal concepts (which do not form a lattice) proposed by Elloumi et al. [9], Li and Zhang [20] discussed the attribute characteristics by restricting them to the regular implication operators which satisfy the transitivity condition, and they presented a deletion algorithm to obtain a δ-reduct (not all δ-reducts).

Compared to the classical formal contexts, very little research has been done on knowledge reduction in formal fuzzy contexts. We are in lack of a general method for calculating knowledge reduction in formal fuzzy contexts parallel to that proposed by Zhang and Wei [41] for classical formal contexts. Based on the idea of variable threshold concept lattices, we propose in this paper a more general method to reduce the size of a formal fuzzy context without changing the structure of the lattice. Specifically, we construct appropriate discernibility matrices and discernibility functions to calculate the attribute (object) reducts in a formal fuzzy context by which all the attribute (object) reducts in the formal fuzzy context are determined. When δ = 1, the proposed reduction approaches can be adapted to the "one-sided fuzzy concept".

The paper is organized as follows. In the next section, we review some basic notions of FCA and fuzzy logic relevant to the discussion throughout the paper. We also recall the variable threshold concepts introduced by Zhang et al. [40]. Section 3 proposes the approaches to attribute reducts and object reducts in formal fuzzy contexts. In Section 4, we define the discernibility matrices and Boolean functions to determine all reducts of a formal fuzzy context. We then conclude the paper with a summary and outline some future direction of research in Section 5.

2. Preliminaries and background on variable threshold concept lattices

To facilitate our discussion, we first briefly recall some relevant notions of FCA and fuzzy logic, and then discuss some properties of the variable threshold concept lattices. Detailed information can be found in [11,1,40].

The basic data set of FCA is a formal context, which is a triple (U, A, I), where U is a set of objects called the universe of discourse, A a set of features or attributes, and I a binary relation between U and A, which is a subset of the Cartesian product $U \times A$, where $(x, a) \in I$ means that object x has attribute a.

A formal concept of (U, A, I) is a pair (X, B) of sets $X \subseteq U$ (called extent) and $B \subseteq A$ (called intent) such that $X = B^*$ and $B = X^*$, where

$$X^* = \{a \in A | \forall x \in X, (x, a) \in I\},$$
$$B^* = \{x \in U | \forall a \in B, (x, a) \in I\}.$$

The set of all formal concepts of (U, A, I) is denoted by $L(U, A, I)$. For any $(X_1, B_1), (X_2, B_2) \in L(U, A, I)$, partial order $\leqslant$ is defined by

$$(X_1, B_1) \leqslant (X_2, B_2) \text{ iff } X_1 \subseteq X_2 \text{ (iff } B_2 \subseteq B_1).$$

$L(U, A, I)$ forms a complete lattice, the so-called concept lattice, in which the infimum and the supremum are given by

$$(X_1, B_1) \wedge (X_2, B_2) = (X_1 \cap X_2, (B_1 \cup B_2)^{**}),$$
$$(X_1, B_1) \vee (X_2, B_2) = ((X_1 \cup X_2)^{**}, B_1 \cap B_2).$$

Definition 1 ([1,2]). A residuated lattice is a structure $\mathbf{L} = (L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ such that

(1) $(L, \vee, \wedge, 0, 1)$ is a lattice with the least element 0 and the greatest element 1;
(2) $(L, \otimes, 1)$ is a commutative monoid (i.e. $\otimes$ is commutative, associative, and $a \otimes 1 = 1 \otimes a = a$ for each $a \in L$);
(3) $\otimes, \rightarrow$ form an adjoint pair, i.e. $x \leqslant y \rightarrow z$ iff $x \otimes y \leqslant z$ hold for all $x, y, z \in L$.

In above definition, join $\vee$ and meet $\wedge$ are supremum and infimum, multiplication $\otimes$ and residuum $\rightarrow$ are (truth functions of) "fuzzy conjunction" and "fuzzy implication" respectively. A common choice of L is a structure with $L = [0, 1]$, $x \vee y = max(x, y)$ and $x \wedge y = min(x, y)$, and with three most important pairs of adjoint operations [1]: the Łukasiewicz one ($x \otimes y = max(x + y - 1, 0)$, $x \rightarrow y = min(1 - x + y, 1)$), Gödel one ($x \otimes y = min(x, y)$, $x \rightarrow y = 1$ if $x \leqslant y$ and $= y$ else), and product one ($x \otimes y = x \cdot y$, $x \rightarrow y = 1$ if $x \leqslant y$ and $= y/x$ else).

A residuated lattice $(L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ is called complete if $(L, \vee, \wedge)$ is a complete lattice. For convenience, we write a residuated lattice $\mathbf{L} = (L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ as $\mathbf{L}$.

In what follows, the complete residuated lattice $(L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ is fixed with the supporting set [0,1], i.e. $L = [0, 1]$. The notion of a residuated lattice provides a very general truth structure for fuzzy logic and fuzzy set theory.

Let $\mathbf{L}$ be a residuated lattice and U a nonempty set. A mapping $A : U \rightarrow \mathbf{L}$ is referred to as a $\mathbf{L}$-set of U, and $A(x)$ is interpreted as the truth degree of the fact "x belongs to A" for any $x \in U$. By L^U we denote the set of all $\mathbf{L}$-sets of U. The empty set $\emptyset$ and the universe set in L^U are denoted by $\overline{0}_U$ and $\overline{1}_U$, respectively. For $X_1, X_2 \in L^U, X_1 \subseteq X_2$ if and only if $X_1(x) \leqslant X_2(x)$ for all $x \in U$. Operations $\wedge$ and $\vee$ on L^U are, respectively, defined as follows:

$$(X_1 \wedge X_2)(x) = X_1(x) \wedge X_2(x),\ (X_1 \vee X_2)(x) = X_1(x) \vee X_2(x),\quad \forall x \in U.$$

Similar to the classical situation, a formal fuzzy context is a triple $(U, A, \widetilde{I})$, where U and A are, respectively, the object set and attribute set, and $\widetilde{I} \in L^{U \times A}$ is a $\mathbf{L}$-fuzzy relation between U and A. The following illustrative example is an extension of the formal fuzzy context in [9].