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Group decision making with intuitionistic fuzzy preference relations

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ABSTRACT

The capability of intuitionistic fuzzy preference relation in representing imprecise or not reliable judgments which exhibit affirmation, negation and hesitation characteristics make it an attractive research area in group decision making. As traditional fuzzy set theory cannot be used to express all the information in a situation as such, its applications are limited. In Zadeh's fuzzy set, the membership degree of an element is defined by a real value, and nonmembership is expressed by a complement of membership. This membership definition actually ignores the decision maker's hesitation in the decision making process. The advantage of Atanassov's intuitionistic fuzzy sets is the capability of representing inevitably imprecise or not totally reliable judgments and the capability of expressing affirmation, negation and hesitation with the help of membership definitions. The consistency of intuitionistic fuzzy preference relations and the priority weights of experts gathered from these preference relations play an important role in group decision making problems in order to reach an accurate decision result. In this paper, we propose a group decision making process with the usage of intuitionistic fuzzy preference relations where we mainly focus our attention on the investigation of consistency of intuitionistic fuzzy preference relations. Initially, we present two different optimization models to minimize the deviations from additive and multiplicative consistency respectively. The optimal deviation values obtained from the model results enable us to improve the consistency of considered preference relations. Then, based on consistent collective preference relations, two mathematical programming models are established to obtain the priority weights, of which the first is a linear programming model considering additive and the second one is a nonlinear model considering multiplicative consistency. Furthermore, a number of numerical illustrations are presented to observe the validity and practicality of the models. Finally, comparative analyses were performed in order to examine the differences between fuzzy and intuitionistic fuzzy preference relations and the results of the analyses showed that the priority vectors and ranking of the alternatives maintained from fuzzy or intuitionistic fuzzy preference relations change significantly.

1. Introduction

Decision making can be considered as the mental processes in which we make a selection among several alternative choices. Making a decision implies that there are alternative choices to be considered, and in such a case we want to choose the one that has the highest probability of success or effectiveness and best fits with our goals, desires, lifestyle or values. In the decision making process, a decision maker (DM) is usually asked to give his/her preferences over alternatives. In this process, preference relations (referred to as pairwise comparison matrices, judgment matrices) help us to explain DM's preference information in decision making problems of several fields. During the last decades, the concept of preference relations has received an increasing attention and several studies have been developed on this subject. In 2007, Xu presented a comprehensive survey of preference relations [54]. In decision making problems, the experts' preferences on decision alternatives are commonly described by multiplicative preference relations [32,33,62,50,53], fuzzy preference relations [29,37,23,7–12,21,22,15,5,24,28,46] or linguistic preference relations [18,19,50,51,13,40,38].

However, in most real life decision making problems, the DMs may not be able to provide his/her preferences for alternatives to a certain degree due to lack of precise or sufficient level of knowledge related to the problem, or the difficulty in explaining explicitly the degree to which one alternative is better than others. In these situations, there is usually a degree of uncertainty in providing their preferences over the alternatives considered, which makes the result of the preference process exhibit the characteristics of affirmation, negation and hesitation [60]. The voting example is an appropriate example of such a case, where "yes", "no" and

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"abstain" votes are possible. Abstention votes may be considered as votes which are "unclassifiable" and indicate the hesitation and indeterminacy of the voter over the alternatives. As traditional fuzzy set theory, introduced by Zadeh [64] cannot be used to express all the information in such a situation, its applications are limited [60]. In Zadeh's fuzzy set, the membership degree of an element is defined by a real value μ, where $0 \leqslant \mu \leqslant 1$ and nonmembership degree is expressed by $1 - \mu$. This expression of membership provides a powerful framework to characterize vagueness and uncertainty [41]. However, the representation of nonmembership as a complement of membership actually ignores the DM's hesitation in the decision making process. In 1986, Atanassov extended Zadeh's fuzzy set, which only assigns a membership degree to each element, and introduced the concept of intuitionistic fuzzy sets which simultaneously consider the degrees of membership and nonmembership with hesitation index [1,2]. The advantage of Atanassov's intuitionistic fuzzy sets is the capability of representing inevitably imprecise or not totally reliable judgments [30] and the capability of expressing affirmation, negation and hesitation with the help of membership definitions.

The consistency of intuitionistic fuzzy preference relations (IFPRs) and the priority weights of experts gathered from these preference relations play an important role in group decision making problems in order to reach an accurate decision result. In the present study, a group decision making model with intuitionistic fuzzy preference relations considering both aggregation of individual preference relations and consistency aspects is proposed. Throughout this study, the consequences of additive consistent and multiplicative consistent IFPRs on priority weights is examined. A linear programming model considering additive consistency and a nonlinear model considering multiplicative consistency has been developed to calculate the priority weights. These models also enable us to improve the consistency of considered preference relations whereby consistent individual preference relations before aggregation can be obtained. Furthermore, some illustrative examples are presented in order to examine the validity and practicality of the developed models. Numerical analyses have shown that although the priority weight vectors of the individual preference relations of the experts differ, the ranking of the individual priority weights do not differ significantly according to the additive consistent or multiplicative consistent intuitionistic fuzzy preference relations. If we derive consistent preference relations (additive or multiplicative consistent), the ranking of the alternatives obtained from collective preference relation or aggregated priority vectors will generally be the same. Additionally, in the current literature, the comparison of the usage of fuzzy and intuitionistic fuzzy preference relations in the group decision making problems has not been investigated. The analysis of the ranking of the alternatives in two cases (fuzzy and intuitionistic fuzzy ones) is an interesting study area. In the present study, the usage of fuzzy and intuitionistic fuzzy preference relations in the group decision making problems is investigated. The differences of fuzzy and intuitionistic fuzzy preference relations, priority vectors and ranking of the alternatives obtained from these preference relations are analyzed.

The results of the numerical illustrations showed that intuitionistic fuzzy preference relations provide more accurate priority vectors and rankings of alternatives by taking into consideration the DMs' affirmation, negation and hesitation with the help of membership definitions.

The remainder of the paper has been organized as follows: In Section 2, a literature review on the subject is presented. In Section 3, some basic concepts about IFPR are explained. In Section 4, the relation between FIPRs and IFPRs and the consistency issues of IFPRs are analyzed. Sections 5 and 6 provide two optimization models to calculate the priority weight of additive consistent and multiplicative consistent collective IFPRs respectively. In Section 7, numerical examples are given to illustrate the validity and practicality of the proposed methods. Section 8 provides comparative analyses of fuzzy and intuitionistic fuzzy preference relations and Section 9 concludes this paper.

2. Literature review

In the literature, Atanassov's intuitionistic fuzzy set theory has been studied by many researchers dealing with decision making concept [3,25,27,26,61,55,56,42–44,16,17,34,39,6]. Szmidt and Kacprzyk [35,36] introduced the definition of the intuitionistic fuzzy preference relation (IFPR). In addition, they also studied the consensus reaching process, and analyzed the extent of agreement in a group of experts. Atanassov et al. [3] proposed an algorithm for solving the multi-person multi-attribute decision making problems, in which the attribute weights are given as exact numerical values and the attribute values are expressed in intuitionistic fuzzy numbers. Li [25] investigated multi-attribute decision making with intuitionistic fuzzy information and established several linear programming models to generate optimal weights for attribute. Lin et al. [27] proposed a new method for handling multiple attribute fuzzy decision making problems, where the characteristics of the alternatives are represented by Atanassov's intuitionistic fuzzy sets. Li et al. [26] presented the fractional programming method for multiple attribute group decision making using Atanassov's intuitionistic fuzzy sets. Xu [56] investigated group decision making problems based on IFPR and incomplete IFPR. He used averaging operators to aggregate intuitionistic preference information and applied score and accuracy functions for the ranking and selection of alternatives.

Priority weight generation from the preference relations is the main issue of group decision making concept. Preference relation presents a common format which provides the opportunity to explain DM's preference information in decision making problems by pairwise comparisons [45]. However, in the process of decision making it is very difficult for a DM to construct a consistent preference relation. Since an inconsistent preference relation may lead to wrong conclusions, priority weight generation methods should take into consideration the consistency of preference relations. Most of the priority weight generation methods in the fuzzy set theory papers in the literature are based on fuzzy interval preference relations (FIPR), introduced by Xu [48]. Xu and Chen [61] proposed a number of linear programming models for deriving the priority weights from various fuzzy interval preference relations considering additive and multiplicative consistency. Genç et al. [15] showed that the consistency and the priority weights can be derived by simple formulas based on interval multiplicative transitivity rather than linear programming models proposed by Xu and Chen [61]. Furthermore, these authors proposed two approaches in order to estimate missing values of an incomplete FIPR. Xu [58] investigated the consistency of fuzzy interval preference relations. Initially he established a quadratic programming model to establish the importance weights of experts. He then proposed two approaches to constructing additive and multiplicative consistent fuzzy interval preference relations. Additionally, he showed the relationship between the consistency of individual FIPRs and the consistency of collective FIPR.

Gau and Buehrer [14] introduced the concept of vague sets (interval valued fuzzy sets) and Bustince and Burillo [4] showed that the notion of vague sets is actually that of intuitionistic fuzzy sets. This argument assists researchers to construct priority weight generation methods based on intuitionistic fuzzy preference relations. Xu [55] defined the concept of additive consistent intuitionistic fuzzy preference relation (IFPR) and established a method for estimating criteria weights from intuitionistic fuzzy preference