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Review

On the use of continuous relative phase: Review of current approaches and outline for a new standard

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ABSTRACT

Background: In this paper we review applications of continuous relative phase and commonly reported methods for calculating the phase angle. Signals with known properties as well as empirical data were used to compare methods for calculating the phase angle.

Findings: Our results suggest that the most valid, robust and intuitive results are obtained from the following steps: 1) centering the amplitude of the original signals around zero, 2) creating analytic signals from the original signals using the Hilbert transform, 3) calculating the phase angle using the analytic signal and 4) calculating the continuous relative phase.

Interpretations: The resulting continuous relative phase values are free of frequency artifacts, a problem associated with most normalization techniques, and the interpretation remains intuitive. We propose these methods for future research using continuous relative phase in studies and analyses of human movement coordination.

1. Introduction

Within sports and health science, the biomechanical study of human movement has many purposes; these include, but are not limited to, rehabilitation, injury prevention and sports performance analysis. A common challenge for all of these domains is simplifying the high-dimensional information available from 2D video analysis, 3D motion capture systems or other modes of kinematic data collection. Dynamical systems theory approaches to movement analysis have gained support in recent years because it provides a theoretical framework for simplifying and working with complex systems (see, e.g. (Kelso, 1995)). Dynamical systems can be composed of many parts interacting and their behavior may often be described by a single low-dimensional term or measure. Most human movements involve a great number of moving parts, all coordinated together, explaining why so many researchers and clinicians have put such effort into modeling the human movement system as a dynamical system (e.g. (Davids et al., 2003; Glazier and Davids, 2009; Stergiou, 2004)). For example, in locomotion the lower extremity segments can be treated as a coupled system and the interaction of the segments acts to effectively displace the body's position during locomotion. By treating the musculoskeletal system as a system evolving over time, rather than focusing on particular events, a much richer description of the interaction of the individual and his environment can be achieved (Barela et al., 2000).

Rosen (1970) is often cited for suggesting that the behavior of a dynamical system can be described by plotting a variable versus its first derivative — these plots are commonly called phase portraits and provide qualitative utility in analyzing human movement (Bartlett and Bussey, 2012; Beek and Beek, 1988). According to (Clark et al., 1993), the phase portraits of the shank and thigh are similar to a limit cycle system — their coordination is cyclic and dissipative and therefore energy must be supplied to continue the behavior. Accordingly, their relation in phase space, or *relative phase*, can describe the dynamic coordination of these variables. Continuous relative phase is a measure, which describes the phase space relation between two segments (modeled as pendula) as it evolves throughout the movement, which makes continuous relative phase an attractive and popular collective variable for inter- and intra-limb coordination.

A central goal in dynamical systems theory is to identify the attractors, or stable states, of the system. Identifying stable states goes beyond simply identifying the common coordinative states for a particular movement; analysis of the variability of continuous relative phase allows one to investigate the stability of the system, or its resiliency to perturbation. Kelso (1995) noted that when coordination is perturbed beyond stability the relative phase pattern will fluctuate, indicated by an increase in variability, before settling on a new stable pattern. Analyses of the variability of continuous relative phase are insightful tools for understanding the dynamics of higher order coordination.

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Therefore, the importance of a valid, robust method for calculating phase angles, to be sure that the signal of interest is measured without contamination from frequency artifacts, should be clear and will be addressed in this paper.

Both the wide ranging applications of continuous relative phase as well as the varying methods used in its calculation warrant an in-depth overview and discussion of its application, calculation and interpretation. This paper provides an overview of the use of continuous relative phase in sport and health science before comparing the approaches that have been taken in the literature for its calculation. We demonstrate the prominent procedures in the literature using synthetic and empirical data and outline what we suggest to be the new methodological standard for continuous relative phase in sports and health science.

2. Calculating continuous relative phase

Continuous relative phase is a new signal generated representing the difference in phase angles of the two original signals. For the calculation of phase angles two different methods have commonly been used in studies of human movement. Firstly, continuous relative phase between two signals can be calculated based on phase portraits (Burgess-Limerick et al., 1993; Hamill et al., 1999) and, secondly, relative phase between two signals can be calculated using analytic signals generated by the Hilbert transform (Lamoth et al., 2009; Palut and Zanone, 2005). In the following two subsections we describe these methods in detail.

2.1. Phase portraits

Studies of human movement coordination are often grounded in dynamical systems theory; therefore, system components can be assigned to a phase space in which each state of the dynamical system is described by certain properties. Pertaining to continuous relative phase analyses, the phase space usually consists of the measured (time dependent) signal $x(t)$ and its velocity $\dot{x}(t)$, the first derivative of the signal. The measured signal used in phase portraits is most often a segment or joint angle, although others have used higher derivatives to construct the phase space (Wagenaar and van Emmerik, 2000). To calculate the phase angle, frequency effects of the phase portrait on the phase angle are reduced by normalization methods.

Before introducing normalization methods we should first distinguish between analyzing sinusoidal signals and non-sinusoidal signals. Sinusoidal (harmonic) signals are signals which can mathematically be described by a sine wave, for example, the signal

$$x(t) = A\sin(\omega t + \psi) + d \tag{1}$$

where ω denotes the frequency, ψ denotes a constant shift along the x-axis, A is a constant describing the magnitude of the amplitude, and d is a constant which describes a shift along the y-axis. Non-sinusoidal (non-harmonic) signals are those which cannot be mathematically described by only a sine wave (such as in Eq. (1)). For each of these types of signals there are some commonly used normalization techniques.

In order to analyze a sinusoidal signal, Fuchs et al. (1996) showed that the phase portrait should be normalized so that the resulting trajectory in phase space is circular and centered around the origin of the phase space. To achieve the circularity they showed that the $\dot{x}(t)$ axis of the signals should be normalized by multiplying the $\dot{x}(t)$ axis by the factor $\frac{1}{\omega}$: the inverse of the signal's frequency. Furthermore, in case a sinusoidal oscillator is described by Eq. (1) with $d \neq 0$ the oscillator must be shifted by $-d$, so that the phase portrait is centered around the origin of the $x\dot{x}$ phase space. This ensures that phase portraits of different sinusoidal signals $x_1(t)$ and $x_2(t)$ are comparable and hence avoid artifacts caused by frequencies and/or different shifts d_1 and d_2. To calculate phase angles, the displacement of sinusoidal data does not need to be normalized because the phase angle ϕ of a sinusoidal oscillator (for simplicity we assume $d = 0$) does not influence the calculation of ϕ

$$\begin{aligned}\phi &= \arctan\left(\frac{\dot{x}(t)}{x(t)}\right) \\ &= \arctan\left(\frac{\omega\ A\ \cos(\omega\ t + \psi)}{A\ \sin(\omega\ t + \psi)}\right) \\ &= \arctan\left(\frac{\omega\ \cos(\omega\ t + \psi)}{\sin(\omega\ t + \psi)}\right).\end{aligned} \tag{2}$$

To analyze non-sinusoidal signals, different normalization methods have been used. The goal of normalizing the data has been to transform the phase portraits in such a way that both displacement of the signal and its first derivative are limited to the range between -1 and 1. In this paper we used the two most frequently used methods (similar to those reported by (Kurz and Stergiou, 2002)). First, normalization is accomplished for any input signal $y(t)$ by the function

$$f(y(t_i)) = \frac{y(t_i)}{\max(|y(t)|)} \tag{3}$$

This technique limits the input signal of the function to either -1 or 1 depending on the maximum absolute value of $y(t)$. This method is often used for velocity normalization because the zero value has qualitative meaning and, arguably, should be preserved. In other words, after normalization the zero value represents the zero value in the original signal. A second normalization technique is based on the function

$$g(y(t_i)) = 2\left(\frac{y(t_i) - \min(y(t))}{\max(y(t)) - \min(y(t))}\right) - 1 \tag{4}$$

This function transforms the original values $y(t)$ in such a way that the minimum value of $g(y(t))$ equals -1 and the maximum value of $g(y(t))$ equals 1. Here the zero value is midway between the maximum and minimum and can, therefore, be arbitrary. Since angle definitions can be arbitrary, the method in Eq. (4) has often been used for normalizing joint or segment angles. We summarize the normalization methods found in the literature as follows:

- Method A uses Eq. (4) to normalize the joint angular displacement and Eq. (3) to normalize the angular velocities (Barela et al., 2000; Burgess-Limerick et al., 1993; Dierks and Davis, 2007; Hamill et al., 1999; Heiderscheit et al., 1999; Hein et al., 2012; Li et al., 1999; Miller et al., 2008, 2010; Stergiou et al., 2001a,b; Yen et al., 2009).
- Method B uses Eq. (4) for both angular displacement and angular velocity normalization (Figueiredo et al., 2012; Haddad et al., 2010; Kwakkel and Wagenaar, 2002; Lamoth et al., 2002; Meyns et al., 2013; Selles et al., 2001; van Emmerik and Wagenaar, 1996).

After normalization, the phase angle of the signal at time t_i is calculated based on the normalized phase portrait (Barela et al., 2000; Li et al., 1999; Peters et al., 2003)

$$\phi(t_i) = \arctan\left(\frac{\dot{x}_{\text{norm}}(t_i)}{x_{\text{norm}}(t_i)}\right) \tag{5}$$

Finally, the continuous relative phase, $\text{crp}(t_i)$, at time t_i between two signals $x_1(t)$ and $x_2(t)$ is calculated as

$$\begin{aligned}\text{crp}(t_i) &= \phi_1(t_i) - \phi_2(t_i) \\ &= \arctan\left(\frac{\dot{x}_{1,\text{norm}}(t_i)x_{2,\text{norm}}(t_i) - \dot{x}_{2,\text{norm}}(t_i)x_{1,\text{norm}}(t_i)}{x_{1,\text{norm}}(t_i)x_{2,\text{norm}}(t_i) + \dot{x}_{1,\text{norm}}(t_i)\dot{x}_{2,\text{norm}}(t_i)}\right).\end{aligned} \tag{6}$$