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Ambiguous games played by players with ambiguity aversion and minimax regret ☆

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ABSTRACT

In real-life strategic interactions, a player's belief about the possible payoffs of a strategy profile is often ambiguous due to limited information, and this ambiguity is not be appropriately captured by the traditional game-theoretic framework. In order to address this issue, we introduce a new game model, called an ambiguous game, which incorporates human cognitive factors of ambiguity aversion and minimax regret. Moreover, we also study how the degrees of ambiguity in beliefs about possible payoffs can influence the solutions of an ambiguous game. In particular, we identify the conditions under which a player should release more or less information to his or her opponents. This result provides some insight on how to manage our private information in an ambiguous game, which helps us obtain a better outcome.

1. Introduction

As a powerful tool for modeling strategic interactions, game theory is widely used in many fields such as artificial intelligence [28,30,37]. In game theory, a strategic (normal form or static) game is the basic model of a strategic situation. It is assumed in such a model that for each player the consequence or payoff of a strategy profile is determinate or precise. However, this assumption about a strategic game seems implausible when ambiguity is present [4,21,35], since for some strategy profiles a player may not be able to determine their precise payoffs due to limited information.

Simply put, "ambiguity" refers to a kind of uncertainty in a situation wherein a decision maker cannot assign a precise probability distribution over the possible consequences of an action [7,16,32]. Therefore, an appropriate model of strategic interactions with ambiguity needs to take into account the following problems.

- The payoffs in a strategic game are indeterminate. In other words, the players should assign a *set* of possible payoffs to each strategy profile due to limited information.
- Beliefs regarding the possible payoffs of each strategy profile are ambiguous. That is to say, the players should assign an *imprecise* probability distribution over the set of the possible payoffs, which represents limited information.
- The model should also investigate how partial information or the imprecise probability distribution can influence the solutions of such a game. This will help players manage private information in the interactive situations involving ambiguity.

The Bayesian game [10] is a well-known model that is used to handle the first issue. Its construction is based on the assumption that a player's belief about other players' types is accurate. This essentially means that the player is assumed to have a precise probability distribution over the set of the types under ambiguity. Another approach to modeling strategic situations with ambiguous payoffs is the fuzzy game [4,15,17,22]. In a fuzzy game the main concern is with fuzzy payoffs. The third issue described above, however, is not taken into consideration by these existing models.

To handle the second problem, some approaches [12,19,23] use the theory of multiple priors (a set of probabilities) to represent players' beliefs, and employ the maximin decision rule to determine a preference ordering. Such a decision rule requires a decision maker to choose an option that maximizes the minimal expected utilities with respect to the set of multiple priors. Based on this idea, these approaches provide some solutions for strategic games with ambiguity. Nevertheless, like Bayesian games and fuzzy games, they do not investigate the relation between partial information and the solutions of the defined game.

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The aim of this paper is to develop an appropriate game-theoretic framework that can deal with the above problems in a strategic interaction with ambiguity. It has been shown in behavioral economics that decision makers are often influenced by ambiguity aversion [2,3,11,18,29] and minimax regret [2,24,25,36] when facing ambiguity. Recently, some researchers [20,21] have attempted to construct a decision-theoretic framework to model decision making under ambiguity. The main tool employed in this framework to represent ambiguity is the well-known Dempster–Shafer (D–S) theory [31], which is a kind of imprecise probability theory and is often applied to knowledge-based systems and decision making under ambiguity [8,13,34]. Moreover, it uses the rule of ambiguity aversion and minimax regret to determine a preference ordering over actions with respect to interval-valued expected utilities. Based on this framework, we propose a new game model called "an ambiguous game".

The main contribution of this paper lies in defining and investigating a new game-theoretic model that can properly handle strategic interactions involving ambiguity. More precisely, the game model proposed here relaxes the rather stringent assumption of strategic games, which requires that for each player the payoff of a strategy profile should be precise. In contrast with a Bayesian game, our game model allows a player's belief concerning the possible consequences or payoffs of each strategy profile to be represented by an imprecise probability. In addition, we identify to what extent the degrees of ambiguity in players' beliefs can influence the solutions of an ambiguous game. It is hoped that our approach can shed light on how to better manage personal information under interactive situations involving ambiguity.

The remainder of this paper is organized as follows. Section 2 briefly reviews some basic concepts and notations of the D–S theory and presents a decision-theoretic framework that takes into account the cognitive factors of ambiguity aversion and minimax regret. Section 3 describes the basics of strategic games with indeterminate payoffs, and then introduces the corresponding solution concept to such games. Section 4 investigates the conditions under which the degrees of ambiguity in beliefs about possible consequences can influence the solutions of an ambiguous game. Section 5 illustrates our approach by considering a scenario of allocating resource. Section 6 discusses some related research on the issue of ambiguity. Finally, Section 7 concludes the paper and outlines some possible directions for future research.

2. Preliminaries

This section first provides a brief review of the D–S theory [31], and then presents a decision model based on this theory.

2.1. Basics of D–S theory

Let us begin by defining the notion of a mass function that plays a central role in the D–S theory.

Definition 1. Let Θ be a frame of discernment, (i.e., the set of states of the world). A function $m: 2^{\Theta} \to [0,1]$ is called a basic probability assignment or a mass function over Θ if $m(\emptyset) = 0$ and $\sum_{A\subseteq\Theta} m(A) = 1$. If $m(A) > 0$, then A is said to be a focal element. A mass function m is called a simple mass function over Θ if $m(A_0) = s,\ m(\Theta) = 1 - s$, where $A_0 \subset \Theta$, and $0 \leqslant s \leqslant 1$. In this case, we call s a focal mass value.

Mass functions can be employed to represent partial information in a reasonable way. In particular, the simple mass function, the basic function in the D–S theory, provides a natural way of modeling various kinds of ambiguity including the cases of total ignorance and partial information. To illustrate this point, let us consider the following example. Suppose that we draw a ball from the urn randomly, where there are 300 balls in total. We know that at least 100 of them are red and the remaining balls are red (r), blue (b) or green (g). How should we model this case where we only have partial information about the proportion of the balls? According to the D–S theory, this case can be represented by a simple mass function: $m(\{r\}) = \frac{1}{3}$, and $m(\Theta) = \frac{2}{3}$, where $\Theta = \{r, b, g\}$. We can thus interpret the focal mass value s as the lower probability of the focal element A_0 [31].

A mass function essentially reflects the degree of ambiguity in available evidence or information. Intuitively, the larger the cardinality of set A, the more ambiguous is a mass function focused on A. As such, we can define the degree of ambiguity of a mass function as follows [5]:

Definition 2. Let m be a mass function over Θ and $|A|$ be the cardinality of set A. Then the degree of ambiguity of m, denoted as δ, is given by

$$\delta = \frac{\sum_{A\subseteq\Theta} m(A)\log_2|A|}{\log_2|\Theta|}. \tag{1}$$

In order to identify the range of the degree of ambiguity in a mass function, let us examine the following two extreme cases. On the one hand, we do not have any information concerning any state of the world, that is, we are completely ignorant (absolute ambiguity). Such a case can be depicted by a mass function m such that $m(A) = 0$ if $A \neq \Theta$, and $m(\Theta) = 1$. According to Definition 2, it is easy to see that $\delta = 1$. On the other hand, we may have sufficient information to obtain a mass function m' such that $m'(\{a\}) = s_1,\ m'(\{b\}) = s_2$, and $m'(\{c\}) = s_3$, where $s_1 + s_2 + s_3 = 1$. In this case, it is obvious that m' is a probability function, and by Formula (1) its degree of ambiguity is given by $\delta' = 0$. Therefore, the degree of ambiguity of a mass function lies in the range $[0, 1]$.

2.2. Expected utility interval

Now consider a decision situation where a decision maker cannot identify a determinate consequence that an action will result in, although he can specify the utilities of those possible consequences. In addition, it may be the case that he cannot even determine the probabilities associated with those possible consequences. According to the D–S theory, the decision maker can represent the ambiguity in terms of a mass function defined over the set of those possible consequences. Formally, such a decision situation can be defined as follows.

Definition 3. A decision problem under ambiguity (or called an ambiguity decision problem) is a 4-tuple of (A, Θ, m, u), where

(i) $A = \{a_1, a_2, \ldots, a_n\}$ is the set of all actions;
(ii) $\Theta = \{c_1, c_2, \ldots, c_k\}$ is the set of all of consequences of actions;
(iii) m is a mass function over the set Θ; and
(iv) u is a utility function, i.e., $u: \Theta \to \mathbb{R}$, where $\mathbb{R}$ is the set of real numbers.

Based on the concept of mass functions, the point-valued expected utility formula can then be extended to the context of ambiguity by defining the notion of an expected utility interval [33]:

Definition 4. Given an ambiguity decision problem (A, Θ, m, u), the expected utility interval of the action $a_i \in A$ is given by $EUI(a_i) = [\underline{U}(a_i), \overline{U}(a_i)]$, where