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On the analytic hierarchy process and decision support based on fuzzy-linguistic preference structures

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ABSTRACT

The Analytic Hierarchy Process (AHP) has received different fuzzy formulations, where two main lines of research can be identified in literature. The most popular one refers to the Extent Analysis Method, which has been subject of recent criticism, among other things, due to a number of misapplications that it may lead to. The other approach refers to the Logarithmic Least Squares Method (LLSM), which offers a constrained optimization approach for estimating fuzzy weights, but fails to generalize the original AHP proposal. The fact remains that the AHP uses linguistic evaluations as input data, where experts value pairs of alternatives/criteria with words, making it essentially fuzzy under the view that words can be represented by fuzzy sets for their respective computation. Hence, reasoning with fuzzy logic is justified by the analytical framework that it offers to design the meaning of words through membership functions and not assume a direct mapping between words and crisp numbers. In this paper we propose the fuzzy representation of linguistic preferences for the AHP, and examine its generalization by means of the fuzzy-linguistic AHP algorithm.

1. Introduction

Decision support methodologies need to take into account the different aspects or dimensions that are relevant for representing and solving common real-life problems, where a given set of alternatives, criteria or some objects of interest have to be evaluated, aggregated and exploited in order to identify priorities and arrive at the most attractive solutions. A popular methodology for establishing such priorities is Saaty's Analytic Hierarchy Process (AHP) [30,34,35]. This technique allows aggregating expert preference judgments made over pairs of objects, which are gathered under the form of a comparison matrix. Traditionally, the AHP elicitation of preferences is based on a valuation scale where one linguistic label agrees with a crisp value or precise number, while the fuzzy AHP allows general type of evaluations taking the form of fuzzy sets.

Two traditional general lines of research can be found in literature concerning the fuzzy methodology for the AHP (see e.g. [28,45,46]). On the one hand, the Extent Analysis Method (EAM) was introduced in 1996 [4], where crisp weights are obtained from the fuzzy comparison matrix. On the other hand, the logarithmic least squares method (LLSM) was firstly proposed in 1983 [20], being later extended and modified (as in [1,45]), estimating fuzzy weights from the respective fuzzy judgments. For a detailed overview on the number of methods found in literature for handling fuzzy comparison matrices based on the AHP, see [45,46].

Different applications for the traditional fuzzy AHP exist, evaluating expert opinions for arriving at decisions that take into account the natural complexity and uncertainty of real-world problems. Many examples on the application of the EAM can be found in literature (see e.g. [3,12,17,18,36,41]), while fewer examples can be found for the LLSM approach (see e.g. [19,43]). Despite its popularity, the EAM has been subject of various criticisms showing that its misapplication may lead to wrong decisions [46,50,51], on the contrary to the modified LLSM-AHP model and its constrained nonlinear optimization model [45] which show better generalization results.

On the other hand, some criticism has been given to the unquestioned fuzzification of the AHP [33]. Here, it is acknowledged that the AHP, as it has been originally proposed [30], makes use of a linguistic valuation scale that enables the use of precise numbers to handle linguistic terms for comparing pairs of objects (as it has been pointed out in [11], the selection of a particular scale is an open problem). For example [24,31,32], consider the traditional valuation scale with crisp numbers from 1 to 9, where the numbers {1, 3, 5, 7, 9} respectively agree with the predicates "not more dominant", "moderately more dominant", "strongly

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more dominant", "very strongly more dominant" and "extremely more dominant", while the numbers in between express compromise between those terms. Hence, the words and predicates that experts use for making their evaluations are understood by means of a one-to-one correspondence with crisp numbers, although it can generally be accepted that linguistic characterizations are less precise than numerical ones.

The justification for using fuzzy logic jointly with the AHP is grounded on the use of linguistic assessments for comparing and valuing the relationship between pairs of objects (see e.g. [5]). In this sense, based on the Computing with Words paradigm [21,29,48,49], fuzzy logic allows examining the way in which computing with words can be developed (see e.g. [8,23,47]), while maintaining the numerical approach of the AHP [30]. Therefore, the objective of this paper is to examine the AHP and the fuzzy LLSM-AHP under a general framework for handling linguistic evaluations, which are represented by fuzzy sets (or even representing more complexity, from both a linguistic and computational viewpoint, by interval or type-2 fuzzy sets [2,10,39,40]). In this way, experts can express their evaluations by means of words and natural language, based on our proposal for *fuzzy-linguistic preference structures*, so a priority order can be assigned on the set of objects according to their estimated fuzzy weights.

In order to do so, this paper is organized as follows. In Section 2, we review the original AHP and the fuzzy LLSM-AHP proposals. In Section 3, linguistic-preference structures are used to represent experts' evaluations under the AHP approach, focusing on the design of the linguistic values by means of fuzzy sets. Section 4 introduces the proposal for decision support based on the linguistic-fuzzy AHP algorithm, and in Section 5, an ordinal ranking procedure for decision support is introduced based on the estimated fuzzy weights. We conclude with some final comments for future research.

2. AHP and fuzzy LLSM-AHP overview

The AHP offers a solid numerical methodology for decision support, where objects, such as criteria or alternatives, are compared in a pairwise manner for estimating their importance through a vector of decision weights [30,34]. In the following, we review the AHP original proposal and the fuzzy LLSM approach to the AHP.

2.1. The analytic hierarchy process

Expert knowledge in the AHP [34] is assumed to exist in the form of crisp numbers, expressing the perception of preference between pairs of objects/criteria $i,j \in X$, where $|X| = n$. Preferences are then introduced into a reciprocal square matrix $\mathrm{M}_{n\times n}$, with elements m_{ij}, such that for every $i,j \in X, m_{ij}^{-1} = m_{ji} = 1/m_{ij}$ and $m_{ii} = 1$.

The methodology of the AHP consists in solving the system for

$$\mathrm{M}w = \lambda_{\max} w, \tag{1}$$

where w is the vector of weights establishing the priorities among criteria and $\lambda_{\max}$ is the largest (principal) eigenvalue of M.

An important aspect of the AHP refers to the estimation of the consistency of M, i.e., the consistency of the experts' judgments. This is done by measuring the deviation of M from its ideal version M$*$, which occurs when its elements are given by $m_{ij} = w_i/w_j$. Due to the fact that the principal eigenvalue of the positive reciprocal matrix $\mathrm{M}_{n\times n}$ is always greater than n, being equal in the case that M = M* holds, the dissimilarity between $\lambda_{\max}$ and n allows measuring the deviation of M from ideal consistency. Hence, the consistency ratio can be used as an indicator to avoid making decisions based on totally random, i.e. *inconsistent*, expert knowledge (a more detailed presentation of the complete AHP methodology can be found in [30]).

Stressing the relevant issue for this paper, regarding the representation of human expert knowledge, notice that although experts express their evaluations in linguistic form, such information is automatically transformed into numbers without further analysis, where words are implicitly understood and handled by crisp/precise numbers. This is the key aspect where fuzzy logic can become a useful tool for enhancing in a descriptive way the AHP and its linguistic approach for decision support.

In the following we review the fuzzy LLSM-AHP [20] and its modified version as presented by Wang et al. [45].

2.2. The fuzzy LLSM-AHP

Based on the AHP, fuzzy numbers have been used to represent experts' judgments, as in the following fuzzy comparison matrix [20],

$$\tilde{\mathbf{M}} = \begin{pmatrix} \widetilde{M}_{1,1} & \widetilde{M}_{1,2} & \cdots & \widetilde{M}_{1,n} \\ \widetilde{M}_{2,1} & \widetilde{M}_{2,2} & \cdots & \widetilde{M}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{M}_{n,1} & \widetilde{M}_{n,2} & \cdots & \widetilde{M}_{n,n} \end{pmatrix} \tag{2}$$

whose elements are given by,

$$\widetilde{M}_{i,j} = \begin{pmatrix} (\mu_{ij1}^L, \mu_{ij1}^K, \mu_{ij1}^U) \\ \vdots \\ (\mu_{ijs}^L, \mu_{ijs}^K, \mu_{ijs}^U) \end{pmatrix}, \tag{3}$$

where $\tilde{m}_{ijq} = (\mu_{ijq}^L, \mu_{ijq}^K, \mu_{ijq}^U)$ represents the fuzzy number given by expert $q \in S, |S| = s$, when comparing criteria i and j. Here, μ^L and μ^U can be considered as the support of the fuzzy number $\tilde{m}$, while μ^K can be taken as its modal value.

In this way, for every $i,j \in X$ and $q \in S$, it holds that

$$\tilde{m}_{ijq}^{-1} = \tilde{m}_{jiq} = (1/\mu_{ijq}^U, 1/\mu_{ijq}^K, 1/\mu_{ijq}^L) \tag{4}$$

and

$$\tilde{m}_{iiq} = (1,1,1). \tag{5}$$

Then, for the above matrix $\tilde{\mathbf{M}}$, there exists an associated triangular fuzzy weight vector

$$\widetilde{W} = (\tilde{w}_1, \ldots, \tilde{w}_n) = ((w_1^L, w_1^K, w_1^U), \ldots, (w_n^L, w_n^K, w_n^U)), \tag{6}$$

such that in the ideal case the absolute consistency condition should hold for every q, such that,

$$\tilde{m}_{ijq} = \tilde{w}_i/\tilde{w}_j = (w_i^L/w_j^U, w_i^K/w_j^K, w_i^U/w_j^L). \tag{7}$$

Under this approach [20], the fuzzy weight vector $\widetilde{W}$ is estimated by solving the following unconstrained minimization problem of the fuzzy logarithmic least squares model,

$$\begin{aligned} \text{Min } J = \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}\sum_{q=1}^{s} & (\ln\ w_i^L - \ln\ w_j^U - \ln\ \mu_{ijq}^L)^2 \\ & + (\ln\ w_i^K - \ln\ w_j^K - \ln\ \mu_{ijq}^K)^2 \\ & + (\ln\ w_i^U - \ln\ w_j^L - \ln\ \mu_{ijq}^U)^2 \end{aligned} \tag{8}$$

such that, after normalization, the estimations for the fuzzy weights are given by [20],

$$\tilde{w}_i \approx \left(\frac{\exp(\ln\ w_i^L)}{\sum_{i=1}^{n} \exp(\ln\ w_i^U)}, \frac{\exp(\ln\ w_i^K)}{\sum_{i=1}^{n} \exp(\ln\ w_i^K)}, \frac{\exp(\ln\ w_i^U)}{\sum_{i=1}^{n} \exp(\ln\ w_i^L)} \right). \tag{9}$$