Knowledge-Based Systems 70 (2014) 231–241

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Stochastic multiple criteria decision making with aspiration level based on prospect stochastic dominance

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ARTICLE INFO

Article history:
Received 10 February 2014
Received in revised form 27 May 2014
Accepted 9 July 2014
Available online 18 July 2014

Keywords:
Stochastic multiple criteria decision making
Aspiration levels
Prospect stochastic dominance
Prospect stochastic dominance degree
PROMETHEE II

ABSTRACT

In order to solve a problem of a discrete stochastic multiple-criteria decision making (MCDM) with aspiration levels, a method on the basis of prospect stochastic dominance is proposed in this paper. The psychological behavior of decision maker, for instance, judgmental distortion, reference dependence and loss aversion, are considered. Based on the concept of prospect theory, aspiration levels are initially took to be the reference points. Definition and related analysis of prospect stochastic dominance degree (PSDD) is given to describe the degree that one alternative dominates another when the prospect stochastic dominance relation for each pair of alternatives with respect to aspiration level is determined. On the basis of the PSDD matrix of alternative pairwise comparisons regarding the aspiration level of every criterion, an overall PSDD matrix is constructed using Choquet integral. Further, according to the concept of the PROMETHEE II, an outramking method is designed to collect the alternatives ranking result. Finally, the effectiveness and applicability of the method proposed are illustrated by two given numerical examples.

1. Introduction

Stochastic multicriteria decision making (MCDM) refers to the problem of alternatives selection with multiple criteria that alternatives consequences regarding criteria are not in crisp numbers, fuzzy numbers or linguistic variables, but in the form of stochastic variables. There are a lot of stochastic MCDM problems in real-world situations [36,51,65,66].

Since Keeney and Raiffa [24] first proposed a method on the basis of theory of multi-attribute utility in order to solve problems of stochastic MCDM, stochastic MCDM problem has attracted more and more attentions from researchers. Over the last decades, some effective methods have been proposed for solving stochastic MCDM problems [8–11,23,25–28,36,52]. For example, D'Avignon and Vincke [8] built a degree of distributive preference on alternate pairwise comparisons regarding each criterion and a degree of distributive outranking among all criteria for stochastic MCDM. Kaya and Kahraman [23], Martel and D'Avignon [36] proposed other outranking methods using confidence indices or preference indices for solving stochastic MCDM problems. Lahdelma et al. [27] developed a stochastic multiobjective acceptability analysis (SMAA) for supporting stochastic MCDM or group decision making analysis.

Stochastic Dominance (SD) rules [15,19,30] are effective tools for decision making that allow us to make a choice among several strategies with limited information regarding the preferences of the decision makers. Since the late 1960s, the view of stochastic dominance has been adopted and developed widely in the fields of finance, economic, statistics, marketing, operation research and agriculture. SD rules have been proposed for supporting stochastic MCDM problem in recent years. Huang et al. [20] proposed multi-attribute stochastic dominance to model world-wide preferences in problems of multi-attribute decision. Zaras and Martel [65] advised to weaken the condition of unanimity and accept attribute condition in a majority, and propose multi-attribute stochastic dominance for a decreased attributes number. Zaras and Martel [37] adopted SD rules to make combining the utility models expected, together with outranking relation models to be possible, in order to gain a solution on solving the stochastic MCDM problem. Zaras [62] combined SD rules with rough set methodology [43] for solving decision making problems under risk. Similar approaches were also used for mixed stochastic and deterministic decision making problems [63,64]. Nowak [39] employed the thresholds concept on the basis of SD and procedures of distillation in ELECTRE-III method [45] in order to collect alternatives ranking. Furthermore, the procedures for the problems of stochastic MCDM according to the interactive approaches and the SD rules were also developed by Nowak [40–42]. Zawisza and Trzpiot [66] integrated

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http://dx.doi.org/10.1016/j.knosys.2014.07.005
0950-7051/© 2014 Published by Elsevier B.V.

the probabilities regarding alternate pairwise comparisons, together with SD rules in order to determine the relations of dominance among the alternatives. Based on estimating inferior, indifferent and superior probabilities regarding alternate pairwise comparisons, Fan et al. [12] used SD rules to determine the ranking of alternatives for SMCDM problem. Besides, methods on the basis of degree of stochastic dominance have also been raised [32,67].

Decision-maker (DM) may have different utility functions, depending on their preferences. The advantage of the SD framework is that very little information on preferences is assumed and any assumptions that are made are general in nature, that is, it is unessential to make complete explicitness on the utility function of the decision-maker. In most situations the construction of the utility function is too difficult because the complete information about an individual's preference is difficult to obtain. Moreover, there is no need to assume some specific statistical distribution of outcomes, such as the normal distribution. The rather than attempting making explicit probability on the expected values of a criterion systematically, using stochastic dominance is not only simpler, but also more informative on the behavior of decision maker when under risk. The disadvantage of SD rules is that for first-order stochastic dominance (FSD), second-order stochastic dominance (SSD), the decreasing absolute risk aversion hypothesis for utility function must be accepted [14,15,19,30], i.e., the SD rules is used in the domain of gains. However, it is inappropriate for all situations. Markowitz [34] observed the occurrence of seeking risk on choices among the negative prospects. Arrow [3], who noticed several economic phenomena, pointed out that utility functions usually illustrate reducing, and sometimes rising total risk aversion. Stiglitz [49] has also raised severe doubts on the hypothesis of the rising total risk aversion. Kahneman and Tversky [22] also conducted experiments to justify this paradox.

For MCDM, decision-maker often has different level of objective or psychological expectation to different criteria that the decision-maker desires to achieve, which is called aspiration level [13,33,39,42]. For instance, decision-maker may have requirements or expectations on the fields [5], such as junction temperature and manufacturing cost, i.e. junction temperature remains lower than 130 °C and manufacturing cost is lower than $70, which is criteria aspiration level, when choosing an alternative power electronic device design. Recently, aspiration levels in MCDM have received more and more attention, and have been investigated by some researcher [5,13,33,38,44,50,58,60], which reflects DM's decision behavior and psychological characteristics [13]. Much empirical evidence [7,22,53,55,59] have shown that the DM's behavior and psychological characteristics, including loss aversion, reference dependence, as well as judgmental distortion of probability of nearly impossible and undoubted outcomes [1,6,21,48], would have an important impact on decision analysis. In practical MCDM problems, the aspiration level is generally regarded as a reference point of DM to criteria [22,67]. Thought the existing methods have made significant contributions to SMCDM analysis, it is seldom considered the aspiration level of criteria [33], that is, the DM's behavior is rarely considered in the existing studies for stochastic MCDM. Therefore, in the situation of considering the DM's psychological behavior or behavioral decision making, it is necessary to develop a new method for solving the stochastic MCDM problem with aspiration levels.

A large number of studies [1,48,6,2,47,57,59,53,54] show that prospect theory [22,55] is the most influential behavioral decision theory which integrates the behavioral principles of decision-maker on the basis of the observations on the process of actual decision making, for instance, diminishing sensitivity, loss aversion and reference dependence. Several studies have already shown that MCDM, which is based on prospect theory, is more accordant with the behavior of decision-maker [6,16,29,46,61]. objective of this paper aims at developing a new method on the basis of integrating prospect theory with stochastic dominance in order to solve the stochastic MCDM problem, together with levels of aspiration. In the proposed method, based on value function of prospect theory, DM's aspiration levels are regarded as the reference points. Prospect stochastic dominance (PSD) relations for pairwise comparisons of alternatives with respect to all criteria are identified using PSD rules [31] where DM's behavior and psychology, such as risk aversion for gains and risk seeking for losses, are considered. Then, according to PSD relation, the corresponding prospect stochastic dominance degree (PSDD) is used in measuring the dominance degree, which an alternative dominates another one. Furthermore, an overall PSDD matrix is constructed. Finally, based on the obtained overall SDD matrix, using PROMETHEE II method [56], a ranking of alternatives is determined.

The rest of this paper is structured as follows: Section 2 gives a brief introduction to value function of prospect theory, and describes prospect stochastic dominance (PSD) rules. Some discussion of PSD rules are given in detail. Section 3 gives the concept and computation formula of prospect stochastic dominance degree (PSDD). Section 4 proposes a method based on the PSDD and PROMETHEE II method for solving the stochastic MCDM problem considering aspiration levels. In Section 5, two numerical examples are given to illustrate the use of the proposed method. Finally, the main characteristics of the proposed method are summarized and highlighted in Section 6 of the paper.

2. Prospect stochastic dominance

2.1. Value function of prospect theory

Prospect theory is a paradigm challenging the expected utility paradigm, which was extended by Tversky and Kahneman [55]. The extended version is known as cumulative prospect theory. The main features of prospect theory are: (i) Investors make decisions based on change of wealth rather than on total wealth, in contrast to what is advocated by expected utility theory. (ii) Investors maximize the expectation of a value function, $V(x)$, where x stands for the change in wealth (rather than total wealth). (iii) Investors subjectively distort probabilities. One of the fundamental components of prospect theory is the S-shaped value function, defined as follows:

$$V(x) = \begin{cases} (x - x_0)^{\alpha}, & x - x_0 \geqslant 0, \\ -\theta(-(x - x_0))^{\beta}, & x - x_0 < 0, \end{cases} \tag{1}$$

where x_0 is the reference point, If the outcome is larger than the reference point, then we perceive the outcome as the gains; otherwise, we perceive the outcome as the losses; α and β denote the curvature of the subjective value function for gains and losses, respectively, where $0 < \alpha < 1$, $0 < \beta < 1$; the values of α and β are larger, and the decision maker is tend to risk; θ is loss-aversion coefficient, shows that the region value power function is more steeper for the losses than for the gains, and $\theta > 1$. The value function describes three important behavioral principles as follows: (i) *Loss aversion*: the DM is more sensitive to losses than to absolutely commensurate gains [2]. Value function in loss domain is steeper than in gain domain, that is, losses looms larger than gains. (ii) *Reference dependence*: the gains and the losses are relative in terms of decision making reference points, and on the same issue, the reference points may be different. (iii) *Diminishing sensitivity*: DM exhibits risk-averse tendency for gains and risk-seeking tendency for losses.

2.2. Prospect stochastic dominance degree

In this section we first briefly review the first-order stochastic dominance (FSD) and second-order stochastic dominance (SSD)