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Optimistic Stackelberg solutions to bilevel linear programming with fuzzy random variable coefficients

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ABSTRACT

In this paper, we consider a kind of bilevel linear programming problem where the coefficients of both objective functions are fuzzy random variables. The purpose of this paper is to develop a computational method for obtaining optimistic Stackelberg solutions to such a problem. Based on $\alpha-$level sets of fuzzy random variables, we first transform the fuzzy random bilevel programming problem into an $\alpha-$stochastic interval bilevel linear programming problem. To minimize the interval objective functions, the order relations which represent the decision maker's preference are defined by the right limit and the center of random interval simultaneously. Using the order relations and expectation optimization, the $\alpha-$stochastic interval bilevel linear programming problem can be converted into a deterministic multiobjective bilevel linear programming problem. According to optimistic anticipation from the upper level decision maker, the optimistic Stackelberg solution is introduced and a computational method is also presented. Finally, several numerical examples are provided to demonstrate the feasibility of the proposed approach.

1. Introduction

The bilevel programming problem is a hierarchical optimization problem with two levels. It is well known that the bilevel programming problem is nonconvex and quite difficult to solve due to its structure. It has been shown that the bilevel programming problem is NP-hard even for the simplest case in which all functions involved are linear [1]. Over the past few decades, this kind of problem has received a great deal of attention and has been successfully applied to a variety of fields, such as network design, transport system, economic, and ecology [2].

Over the decades, the vast majority of research on the bilevel programming problem assumes that all parameters involved in the objective functions and constraints are deterministic. However, in many situations, there are a variety of uncertainties in real-life bilevel decision making problems. Consider a single-period product making decision in which the manufacturer relies on the retailer to sell his/her products. The manufacturer makes a decision first and thereafter the retailer chooses a strategy according to the manufacturer's action. The objectives of the manufacturer and the retailer are to maximize their benefits of product sale. Such a decision making situation can be described by a bilevel programming problem. In practice, the product sale depends upon many factors such as market prices, materials, operation, etc. These factors are fluctuating due to uncertain environments and difficult to estimate precisely. Clearly, the profit sale is also uncertain. Therefore, there is an increasing demand to dealing with not only the bilevel programming but also the uncertainty in decision making. For this purpose, fuzzy or stochastic approaches can be used, and correspondingly the fuzzy bilevel programming and the stochastic bilevel programming have been individually proposed.

In the fuzzy bilevel programming problem, the parameters either in the objective functions or in the constraints are viewed as fuzzy sets and their membership functions are assumed to be known. Sakawa et al. [3] first considered the fuzzy bilevel programming problem and introduced a fuzzy programming method to solve it based on the definition of optimal solution for bilevel programming proposed by Bard [1]. According to Bard's bilevel linear programming theory, it could not well solve a bilevel linear programming problem when the upper level's constraint functions are of arbitrary linear form. To overcome the limitation, Zhang and Lu [4] used the new definition of optimal solution for the bilevel linear programming problem and designed a fuzzy number based Kuhn-Tucker approach to deal with the fuzzy bilevel programming problem. Subsequently, Zhang and Lu [5] presented a fuzzy bilevel decision making model for a general logistics planning problem

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and developed a fuzzy number based Kth-best approach to find an optimal solution for the proposed problem. Zhang et al. [6] proposed the fuzzy Kuhn-Tucker approach, the fuzzy Kth-best approach and the fuzzy branch-and-bound approach to handle the fuzzy bilevel programming problem by applying fuzzy set techniques. In addition, some of the studies in the direction of solving fuzzy multi-objective bilevel optimization problems are conducted [7,8].

On the other hand, in the stochastic bilevel programming problem, the parameters are viewed as random variables and their probability distributions are assumed to be known. Nishizaki et al. [9] used the expectation and variance models to obtain Stackelberg solutions of the multi-objective bilevel programming problem with random variable coefficients. Based on the fractile criterion optimization model, Sakawa and Katagiri [10] developed an interactive fuzzy programming approach for deriving satisfactory solutions of the stochastic bilevel programming problem. Besides, Kosuch et al. [11] studied a stochastic bilevel problem with probabilistic knapsack constraints. They transformed the initial problem into an equivalent quadratic problem and applied an iterative approach to find upper bounds on the original stochastic bilevel problem.

It is worth recalling that the aforementioned bilevel programming problem under uncertainty focuses on either fuzziness or randomness. However, it is significant to realize that real-world decision making situations generally involve both fuzziness and randomness. This kind of combination creates a great challenge for researchers to develop efficient methods to deal with the uncertain environment using both fuzziness and randomness in the bilevel programming problem. So far there have been relatively scarce studies as for the bilevel programming problem under fuzzy random environments. In the case of cooperation, Sakawa and Matsui [12] proposed an interactive fuzzy programming approach to obtain a satisfactory solution of the fuzzy random bilevel programming problem by using $\alpha-$level sets of fuzzy random variables and fractile criterion optimization. Sakawa and Matsui [13] transformed the fuzzy random bilevel programming problem into a deterministic one and developed a computational approach to obtain satisfactory solutions by introducing fuzzy goals together with possibility measures.

The purpose of this paper is to deal with the bilevel linear programming problem with fuzzy random coefficients in both objective functions. Several research works dealing with this kind of problem in the case of noncooperation have been published in recent years [14,15]. Recently, Sakawa and Katagiri [16] transformed the fuzzy random bilevel programming problem into an $\alpha-$stochastic bilevel linear programming problem by using $\alpha-$level sets of fuzzy random variables. Then fuzzy goals for each of the objective functions in the transformed problem were introduced to reflect vagueness of judgments of decision makers. A computational method was proposed to derive Stackelberg solutions for the fuzzy random bilevel programming problem through fractile criterion optimization. Making using of the same transformed processes in [16], Sakawa et al. [17] developed a computational approach to solve the fuzzy random bilevel programming by applying probability maximization.

In these two studies, membership functions are introduced to represent the fuzzy goals for each of the objective functions in the transformed $\alpha-$stochastic bilevel programming problem. On one hand, linear membership functions can be usually used because they have very good properties and are very easy to manipulate. On the other hand, nonlinear membership functions are more realistic and flexible than the linear ones and are usually used for the practical problems, although the complexity of the problem could increase. In general, different membership functions can be used to represent the same uncertain goals. However, different membership functions may lead to different results. This leads to a problem: which one of membership functions should be chosen? To address these issues, interval programming is a good alternative to handle this kind of uncertainty. In fact, an interval number is to only require the bounds of the uncertain coefficients, not necessarily knowing the membership functions.

The main contributions of the study are as follows. We propose an interval programming approach to deal with the fuzzy random bilevel programming problem. In order to do so, we first transform the fuzzy random bilevel programming problem into an $\alpha-$stochastic interval bilevel linear programming problem based on $\alpha-$level sets. The order relations which represent the decision maker's preference are introduced by the right limit and the center of random interval simultaneously. Second, we develop a new decision making model in which the $\alpha-$stochastic interval bilevel linear programming problem can be transformed into a deterministic multiobjective bilevel linear programming problem by using the order relations together with expectation optimization. Third, according to optimistic anticipation from the upper level decision maker [18,9], the optimistic Stackelberg solution is defined and a computational method is presented by means of the Kth best algorithm. The solution obtained by our approach can reflect the decision maker's preference with the right limit and the center of the uncertainty objective function.

This paper is organized as follows. Section 2 recalls basic definitions and preliminary results related to fuzzy random variables. In Section 3, we introduce the bilevel linear programming problem with fuzzy random coefficients. In Section 4, an optimization approach is presented to transform the original problem into a multiobjective bilevel optimization problem, and a computational method for obtaining optimistic Stackelberg solutions is proposed based on optimistic anticipation from the upper level decision maker. Section 5 gives several numerical examples to demonstrate the feasibility of the proposed approach. Section 6 makes the conclusion remarks.

2. Preliminaries

We first recall some basic definitions and results which will be needed in the following sections.

2.1. Interval number

Let R denote the set of all real numbers.

An ordered pair in a bracket defines an interval as

$$c = [c^-, c^+] = \{x : c^- \leqslant x \leqslant c^+, \quad x \in R\},$$

where c^-, c^+ are called the lower and upper bounds of c, respectively.

In interval mathematics, an order relation plays a significant role in ranking interval numbers. We do not say that an interval number is larger than another, instead, we usually say that an interval number is better than another. For the minimization problem, we use the following order relations by Chanas and Kuchta [19].

Definition 1. Let $c_1 = [c_1^-, c_1^+]$ and $c_2 = [c_2^-, c_2^+]$ be two intervals. The order relations $\preceq$ and $\prec$ between c_1 and c_2 are defined as

(1) $c_1 \preceq c_2$ if and only if $c_1^+ \leqslant c_2^+$ and $\frac{c_1^- + c_1^+}{2} \leqslant \frac{c_2^- + c_2^+}{2}$;
(2) $c_1 \prec c_2$ if and only if $c_1 \preceq c_2$ and $c_1 \neq c_2$.

2.2. Fuzzy random variable

We first give some basic concepts of fuzzy numbers as follows.