Knowledge-Based Systems 59 (2014) 9–20

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Robust evidential reasoning approach with unknown attribute weights

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ARTICLE INFO

Article history:
Received 18 January 2013
Received in revised form 26 January 2014
Accepted 31 January 2014
Available online 10 February 2014

Keywords:
Multiple attribute decision making
Evidential reasoning approach
Robust decision
Unknown attribute weights
Incompatibility among alternatives

ABSTRACT

In multiple attribute decision making (MADM), different attribute weights may generate different solutions, which means that attribute weights significantly influence solutions. When there is a lack of sufficient data, knowledge, and experience for a decision maker to generate attribute weights, the decision maker may expect to find the most satisfactory solution based on unknown attribute weights called a robust solution in this study. To generate such a solution, this paper proposes a robust evidential reasoning (ER) approach to compare alternatives by measuring their robustness with respect to attribute weights in the ER context. Alternatives that can become the best with the support of one or more sets of attribute weights are firstly identified. The measurement of robustness of each identified alternative from two perspectives, i.e., the optimal situation of the alternative and the insensitivity of the alternative to a variation in attribute weights is then presented. The procedure of the proposed approach is described based on the combination of such identification of alternatives and the measurement of their robustness. A problem of car performance assessment is investigated to show that the proposed approach can effectively produce a robust solution to a MADM problem with unknown attribute weights.

1. Introduction

With a view to solving a multiple attribute decision making (MADM) problem, the assessments of alternatives on each attribute are usually aggregated after being weighted by attribute weights. The aggregated assessments of alternatives are then used to generate a solution to the MADM problem. Different attribute weights may create different solutions to the problem. As a result, attribute weights significantly influence solutions to the problem.

In literature, there are three categories of methods to determine attribute weights, comprising subjective, objective, and hybrid methods [40]. Subjective methods use the subjective preference of a decision maker to determine attribute weights (e.g., [2,8,13,21,22,28–31,34,36–38,59]). Differently, objective methods use a decision matrix to determine attribute weights (e.g., [4,5,7,9,10,35,40,45]). The subjective preference of a decision maker and a decision matrix are synthetically employed in hybrid methods to determine attribute weights (e.g., [12,25,41]). However, different subjective methods may elicit different attribute weights. There is no single method that can guarantee more accurate attribute weights and further more satisfactory solutions than others [9,10]. Different objective methods are designed on different principles, such as the principle of maximum contrast [9,10] and the combination principle of maximum contrast and minimum correlation [40]. When a decision maker has a lack of sufficient data, knowledge, and experience, he or she will be unsure about which principle is the best to generate the most appropriate attribute weights and further the most satisfactory solution. To guarantee the most satisfactory solution for the decision maker in this situation, all attribute weights in a feasible or predefined weight space rather than a set of attribute weights generated by a specific method should be considered. More specifically, the decision maker would prefer one alternative supported by more sets of attribute weights to be the best to the others in the satisfactory solution. Such a solution is called a robust solution based on unknown attribute weights in this study.

To generate a robust solution, except attribute weights, the assessments of a decision maker need to be flexibly modeled. In a real situation where data for assessing alternatives against criteria are partially or completely unavailable, or the knowledge of the decision maker for alternative evaluation is not sufficient, the decision maker is more likely to give uncertain (or imprecise) assessments. To model various kinds of uncertainties such as

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http://dx.doi.org/10.1016/j.knosys.2014.01.024
0950-7051/

ignorance, fuzziness, interval data, and interval belief degrees in a unified format, the evidential reasoning (ER) approach was developed in the 1990s. It has been under development in recent years [6,19,44,50,54–56] to uniformly solve uncertain MADM problems. In particular, precise and interval numbers can be handled in the ER approach by converting them into distributed assessments [44], similar to the conversion of machine measurements into subjective assessments in the evaluation of new product development [24]. The above analysis shows that the extension of the ER approach to model and solve uncertain MADM problems with unknown attribute weights is a new and significant exploration, which is different from most existing MADM methods (e.g., [1,3,11,23,49,58]).

For this purpose, we firstly identify which alternatives can become the best with the support of at least one set of attribute weights. Then, the robustness of each identified alternative is measured from two perspectives, i.e., the optimal situation of the alternative and the insensitivity of the alternative to a variation in attribute weights, which is used to compare the identified alternatives. The iterations of the two steps are intended to develop a robust ER (RER) approach to generate a robust solution in the ER context.

The main contributions of this paper include the following: (1) the identification of alternatives that can become the best based on unknown attribute weights; (2) the measurement of robustness of each identified alternative from two perspectives; and (3) the development of the RER approach.

The rest of this paper is organized as follows. Section 2 presents the preliminaries related to the RER approach. Section 3 introduces the RER approach. Section 4 presents an investigation regarding car performance assessment to demonstrate the applicability and validity of the RER approach. Section 5 compares the RER approach with the ER approach and other objective methods of determining attribute weights to reveal the advantages of generating a robust solution. Finally, this paper is concluded in Section 6.

2. Preliminaries

2.1. ER approach to solving MADM problems

For the convenience of describing the RER approach, in the following we introduce basic notations and the solution process of the ER approach.

Suppose that a MADM problem has M alternatives a_l ($l = 1,\ldots,M$) and L attributes e_i ($i = 1,\ldots,L$). The relative weights of the L attributes are denoted by $w = (w_1, w_2, \ldots, w_L)$ such that $0 \leqslant w_i \leqslant 1$ and $\sum_{i=1}^{L} w_i = 1$. Assume that $\Omega = \{H_1, H_2, \ldots, H_N\}$ denotes a set of assessment grades. The M alternatives are assessed on the L attributes using H_n ($n = 1,\ldots,N$) in the ER approach. If an alternative a_l is assessed to a grade H_n on an attribute e_i with a belief degree of $\beta_{n,i}(a_l)$, the assessment can be expressed by a belief distribution $B(e_i(a_l)) = \{(H_n, \beta_{n,i}(a_l)), n = 1,\ldots,N; (\Omega, \beta_{\Omega,i}(a_l))\}$, where $\beta_{n,i}(a_l) \geqslant 0$, $\sum_{n=1}^{N} \beta_{n,i}(a_l) \leqslant 1$, and $\beta_{\Omega,i}(a_l) = 1 - \sum_{n=1}^{N} \beta_{n,i}(a_l)$ denotes the degree of global ignorance.

To generate a solution in the ER approach, the assessments $B(e_i(a_l))$ ($i = 1,\ldots,L$, $l = 1,\ldots,M$) are aggregated as $B(y(a_l)) = \{(H_n, \beta_n(a_l)), n = 1,\ldots,N; (\Omega, \beta_\Omega(a_l))\}$ ($l = 1,\ldots,M$) using the analytical algorithm [43], where $\beta_\Omega(a_l)$ denotes the degree of aggregated global ignorance. For the convenience of comparing the M alternatives in the ER approach, $B(y(a_l))$ ($l = 1,\ldots,M$) is combined with the utilities of grades $u(H_n)$ ($n = 1,\ldots,N$) such that $0 = u(H_1) < u(H_2) < \cdots < u(H_N) = 1$ to produce the minimum and maximum expected utilities of the alternative a_l ($l = 1,\ldots,M$), i.e., $u_{\min}(a_l) = \sum_{n=2}^{N} \beta_n(a_l)u(H_n) + (\beta_1(a_l) + \beta_\Omega(a_l))u(H_1)$ and $u_{\max}(a_l) = \sum_{n=1}^{N-1} \beta_n(a_l)u(H_n) + (\beta_N(a_l) + \beta_\Omega(a_l))u(H_N)$. The maximal regret of the alternative a_l ($l = 1,\ldots,M$) is then calculated as $R(a_l) = \max\{0, \max_{j \neq l}\{u_{\max}(a_j)\} - u_{\min}(a_l)\}$. Finally, a rank-order of the M alternatives as a solution to the MADM problem is generated by means of $R(a_l)$ ($l = 1,\ldots,M$) and the minimax regret approach (MRA) [44]. Details regarding the MRA can be found in [44].

2.2. Compatibility between two assessments

The assessments $B(e_i(a_l))$ ($i = 1,\ldots,L$, $l = 1,\ldots,M$) are quasi-Bayesian belief structures (BSs), as presented above, so a compatibility measure between two BSs can be used to measure the compatibility between two assessments. The compatibility measure is given as follows.

Definition 1. Let m be a BS on $\Omega = \{H_1, H_2, \ldots, H_N\}$. Its associated pignistic probability function $BetP(m)$ in the transferred belief model is defined as

$$BetP(m)(w) = \sum_{A \subseteq \Omega, w \in A} \frac{1}{|A|} \cdot \frac{m(A)}{1 - m(\varnothing)}, \quad m(\varnothing) \neq 1,$$

where w can be $H_1, H_2, \ldots,$ or H_N, and $|A|$ is the cardinality of a subset A.

$BetP(m)$ can be extended as a function on 2^Ω, i.e.,

$$BetP(m)(A) = \sum_{B \subseteq \Omega} \frac{|A \cap B|}{|B|} \cdot \frac{m(B)}{1 - m(\varnothing)}, \quad \forall A \subseteq \Omega.$$

The transformation from m to $BetP(m)$ is named as the pignistic transformation.

Definition 2. Let m_1 and m_2 be two BSs on $\Omega = \{H_1, H_2, \ldots, H_N\}$, and $BetP_1$ and $BetP_2$ be their associated pignistic probability functions, respectively. Then

$$difBetP(m_1, m_2) = \max_{A \subseteq \Omega}(|BetP_1(A) - BetP_2(A)|)$$

is called the distance between betting commitments of the two BSs.

Definition 3. Let m_1 and m_2 be two BSs on $\Omega = \{H_1, H_2, \ldots, H_N\}$, and $BetP_1$ and $BetP_2$ be their associated pignistic probability functions, respectively. Suppose $\bar{A} = \{w | w \in \Omega, BetP_1(w) = BetP_2(w) > 0\}$ where $\bar{A}$ is a subset of Ω, then $EP(m_1) = \bar{A}$ and $EP(m_2) = \bar{A}$ are called all equal-pignistic-valued elements of m_1 and m_2, respectively.

Definition 4. Let m_1 and m_2 be two BSs on $\Omega = \{H_1, H_2, \ldots, H_N\}$, $BetP_1$ and $BetP_2$ respectively be their associated pignistic probability functions, $BetPR_1 = \{BetP_1(H_1), \ldots, BetP_1(H_N)\}$ and $BetPR_2 = \{BetP_2(H_1), \ldots, BetP_2(H_N)\}$ respectively be their pignistic transformation results, $EP(m_1)$ and $EP(m_2)$ respectively be their equal-pignistic-valued elements, and $pm_\oplus(\varnothing)(m_1, m_2)$ be the mass of uncommitted belief when combining $BetPR_1$ and $BetPR_2$ with Dempster's rule including no contribution completely from $EP(m_1)$ and $EP(m_2)$. Then, $pm_\oplus(\varnothing)(m_1, m_2)$ is defined as

$$pm_\oplus(\varnothing)(m_1, m_2) = \sum_{w_1 \in \Omega, w_2 \in \Omega, w_1 \cap w_2 = \varnothing} BetPR_1(w_1)BetPR_2(w_2) - \sum_{w_1 \in EP(m_1), w_2 \in EP(m_2), w_1 \cap w_2 = \varnothing} BetPR_1(w_1)BetPR_2(w_2).$$

Definition 5. Let m_1 and m_2 be two BSs, $pm_\oplus(\varnothing)(m_1, m_2)$ be the mass of uncommitted belief as described in Definition 4, and $difBetP(m_1, m_2)$ be the distance between their betting commitments as described in Definition 2. Then, a compatibility measure between m_1 and m_2 is defined as