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Fuzzy analytic hierarchy process with interval type-2 fuzzy sets

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ABSTRACT

The membership functions of type-1 fuzzy sets have no uncertainty associated with it. While excessive arithmetic operations are needed with type-2 fuzzy sets with respect to type-1's, type-2 fuzzy sets generalize type-1 fuzzy sets and systems so that more uncertainty for defining membership functions can be handled. A type-2 fuzzy set lets us incorporate the uncertainty of membership functions into the fuzzy set theory. Some fuzzy multicriteria methods have recently been extended by using type-2 fuzzy sets. Analytic Hierarchy Process (AHP) is a widely used multicriteria method that can take into account various and conflicting criteria at the same time. Our objective is to develop an interval type-2 fuzzy AHP method together with a new ranking method for type-2 fuzzy sets. We apply the proposed method to a supplier selection problem.

1. Introduction

In the traditional formulation of multicriteria decision making (MCDM) problems human's judgments are represented as exact numbers. However, in many practical cases, the data may be imprecise, or the decision makers might be unable to assign exact numerical values to the evaluation. Since some of the evaluation criteria are subjective and qualitative in nature, it is very difficult for the decision maker to express the preferences using exact numerical values [20]. The conventional MCDM approaches tend to be less effective in dealing with the imprecise or vague nature of the linguistic assessments [13]. Analytic Hierarchy Process (AHP) which is one of the most used MCDM approaches is a structured multicriteria technique for organizing and analyzing complex decisions including many conflicting criteria. In the literature fuzzy AHP methods based on type-1 fuzzy sets exist. The fuzzy AHP technique can be viewed as an advanced analytical method developed from the traditional AHP. Despite the convenience of AHP in handling both quantitative and qualitative criteria of multicriteria decision making problems based on decision makers' judgments, fuzziness and vagueness existing in many decision-making problems may cause to the imprecise judgments of decision makers in conventional AHP approaches [7].

In type-1 fuzzy sets, each element has a degree of membership which is described with a membership function valued in the interval $[0,1]$ [21]. The concept of a type-2 fuzzy set was introduced by Zadeh [22] as an extension of the concept of an ordinary fuzzy set called a type-1 fuzzy set. Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets; they are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set; hence, they are useful for incorporating linguistic uncertainties, e.g., the words that are used in linguistic knowledge can mean different things to different people [14]. While the membership functions of type-1 fuzzy sets are two-dimensional, the membership functions of type-2 fuzzy sets are three-dimensional. It is the new third-dimension that provides additional degrees of freedom that make it possible to directly model uncertainties.

An interval type-2 fuzzy set is a special case of a generalized type-2 fuzzy set. Since generalized type-2 fuzzy sets require complex and immense computational burdensome operations, the wide spread application of generalized type-2 fuzzy systems has not occurred. Interval type-2 fuzzy sets are the most commonly used type-2 fuzzy sets because of their simplicity and reduced computational effort with respect to general type-2 fuzzy sets. For this reason, we used interval type-2 fuzzy sets.

In this paper, an interval type-2 fuzzy AHP method is developed and presented into the literature for the first time. The linguistic scale of fuzzy AHP is expressed in a more detailed and flexible way by interval type-2 fuzzy sets. New defuzzification methods for both triangular and trapezoidal type-2 fuzzy sets are also incorporated into the developed method.

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The rest of the paper is organized as follows. Section 2 presents the basics of interval type-2 fuzzy sets. Section 3 gives defuzzification methods including two new methods called DTriT and DTraT. Section 4 includes our proposed interval type-2 fuzzy AHP method. Section 5 gives an illustrative application of the proposed method. Finally Section 6 gives the conclusions.

2. Interval type-2 fuzzy sets

In this section, some definitions of type-2 fuzzy sets and interval type-2 fuzzy sets are briefly explained [17].

A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$, shown as follows [22]:

$$\tilde{\tilde{A}} = \left\{ \left((x,u), \mu_{\tilde{\tilde{A}}}(x,u) \right) \middle| \ \forall x \in X, \ \forall u \in J_x \subseteq [0,1], \ 0 \leqslant \mu_{\tilde{\tilde{A}}}(x,u) \leqslant 1 \right\}, \tag{1}$$

where J_x denotes an interval $[0,1]$. The type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as follows [17]:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u) \Big/ (x,u), \tag{2}$$

where $J_x \subseteq [0,1]$ and $\iint$ denote union over all admissible x and u.

Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\tilde{\tilde{A}}$ is called an interval type-2 fuzzy set [3]. An interval type-2 fuzzy set $\tilde{\tilde{A}}$ can be regarded as a special case of a type-2 fuzzy set, represented as follows [17]:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_x} 1/(x,u), \tag{3}$$

where $J_x \subseteq [0,1]$.

Arithmetic operations with trapezoidal interval type-2 fuzzy sets are given in the following.

Definition 2.1. The upper and lower membership functions of an interval type-2 fuzzy set are type-1 membership functions.

A trapezoidal interval type-2 fuzzy set is illustrated a $\tilde{\tilde{A}}_i = \left(\tilde{A}_i^U; \tilde{A}_i^L\right) = \left(\left(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1\left(\tilde{A}_i^U\right), H_2\left(\tilde{A}_i^U\right) \right), \left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1\left(\tilde{A}_i^L\right), H_2\left(\tilde{A}_i^L\right) \right) \right)$ where $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L$ and a_{i4}^L are the references points of the interval type-2 fuzzy set $\tilde{\tilde{A}}_i, H_j\left(\tilde{A}_i^U\right)$; denotes the membership value of the element $a_{j(j+1)}^U$ in the upper trapezoidal membership function $\left(\tilde{A}_i^U\right), 1 \leqslant j \leqslant 2, H_j\left(\tilde{A}_i^L\right)$ denotes the membership value of the element $a_{j(j+1)}^L$ in the lower trapezoidal membership function $\tilde{A}_i^L, 1 \leqslant j \leqslant 2, H_1\left(\tilde{A}_i^U\right) \in [0,1], H_2\left(\tilde{A}_i^U\right) \in [0,1], H_1\left(\tilde{A}_i^L\right) \in [0,1], H_2\left(\tilde{A}_i^L\right) \in [0,1]$ and $1 \leqslant i \leqslant n$ [8].

Definition 2.2. The addition operation between the trapezoidal interval type-2 fuzzy sets $\tilde{\tilde{A}}_1 = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right) \right), \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L, H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right) \right) \right)$ and $\tilde{\tilde{A}}_2 = \left(\left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\tilde{A}_2^U\right), H_2\left(\tilde{A}_2^U\right) \right), \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\tilde{A}_2^L\right), H_2\left(\tilde{A}_2^L\right) \right) \right)$ is defined as follows [8]:

$$\begin{aligned}\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = &((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U;\\ &\min\left(H_1\left(\tilde{A}_1^U\right); H_1\left(\tilde{A}_2^U\right)\right), \min\left(H_2\left(\tilde{A}_1^U\right); H_2\left(\tilde{A}_2^U\right)\right)),\\ &(a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L;\\ &\min\left(H_1\left(\tilde{A}_1^L\right); H_1\left(\tilde{A}_2^L\right)\right), \min\left(H_2\left(\tilde{A}_1^L\right); H_2\left(\tilde{A}_2^L\right)\right)))\end{aligned} \tag{4}$$

Definition 2.3. The subtraction operation between the trapezoidal interval type-2 fuzzy sets $\tilde{\tilde{A}}_1 = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right) \right), \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L, H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right) \right) \right)$ and $\tilde{\tilde{A}}_2 = \left(\left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\tilde{A}_2^U\right), H_2\left(\tilde{A}_2^U\right) \right), \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\tilde{A}_2^L\right), H_2\left(\tilde{A}_2^L\right) \right) \right)$ is defined as follows [8]:

$$\begin{aligned}\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 = &((a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U;\\ &\min\left(H_1\left(\tilde{A}_1^U\right); H_1\left(\tilde{A}_2^U\right)\right), \min\left(H_2\left(\tilde{A}_1^U\right); H_2\left(\tilde{A}_2^U\right)\right)),\\ &(a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L;\\ &\min\left(H_1\left(\tilde{A}_1^L\right); H_1\left(\tilde{A}_2^L\right)\right), \min\left(H_2\left(\tilde{A}_1^L\right); H_2\left(\tilde{A}_2^L\right)\right)))\end{aligned} \tag{5}$$

Definition 2.4. The multiplication operation between the trapezoidal interval type-2 fuzzy sets $\tilde{\tilde{A}}_1 = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right) \right), \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L, H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right) \right) \right)$ and $\tilde{\tilde{A}}_2 = \left(\left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\tilde{A}_2^U\right), H_2\left(\tilde{A}_2^U\right) \right), \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\tilde{A}_2^L\right), H_2\left(\tilde{A}_2^L\right) \right) \right)$ is defined as follows [8]:

$$\begin{aligned}\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 \cong &((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U;\\ &\min\left(H_1\left(\tilde{A}_1^U\right); H_1\left(\tilde{A}_2^U\right)\right), \min\left(H_2\left(\tilde{A}_1^U\right); H_2\left(\tilde{A}_2^U\right)\right)),\\ &(a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L;\\ &\min\left(H_1\left(\tilde{A}_1^L\right); H_1\left(\tilde{A}_2^L\right)\right), \min\left(H_2\left(\tilde{A}_1^L\right); H_2\left(\tilde{A}_2^L\right)\right)))\end{aligned} \tag{6}$$

Definition 2.5. The arithmetic operations between the trapezoidal interval type-2 fuzzy sets $\tilde{\tilde{A}}_1 = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right) \right), \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L, H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right) \right) \right)$ and the crisp value k is defined as follows [8]:

$$\begin{aligned}k\tilde{\tilde{A}}_1 = &\left((k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U); H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right), \right.\\ &\left.\left(k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L; H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right) \right) \right)\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\tilde{\tilde{A}}_1}{k} = &\left(\left(\frac{1}{k} \times a_{11}^U, \frac{1}{k} \times a_{12}^U, \frac{1}{k} \times a_{13}^U, \frac{1}{k} \times a_{14}^U \right); H_1\left(\tilde{A}_1^U\right), H_2\left(\tilde{A}_1^U\right), \right.\\ &\left.\left(\frac{1}{k} \times a_{11}^L, \frac{1}{k} \times a_{12}^L, \frac{1}{k} \times a_{13}^L, \frac{1}{k} \times a_{14}^L; H_1\left(\tilde{A}_1^L\right), H_2\left(\tilde{A}_1^L\right) \right) \right)\end{aligned} \tag{8}$$

where $k > 0$.

3. Defuzzification methods for type-2 fuzzy sets

Defuzzification of a type-2 fuzzy set consists of two steps. In the first step, a type-2 fuzzy set is determined as a type-1 fuzzy set by using the type reduction process. Then one of the defuzzification methods for ordinary (type-1) fuzzy sets is used to find the equivalence of the type-2 fuzzy set [14]. There are a lot of type reduction methods proposed in the literature. In the following the most used