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A social network analysis trust–consensus based approach to group decision-making problems with interval-valued fuzzy reciprocal preference relations

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ABSTRACT

A social network analysis (SNA) trust–consensus based group decision making model with interval-valued fuzzy reciprocal preference relation (IFRPR) is investigated. The main novelty of this model is that it determines the importance degree of experts by combining two reliable resources: trust degree (TD) and consensus level (CL). To do that, an interval-valued fuzzy SNA methodology to represent and model trust relationship between experts and to compute the trust degree of each expert is developed. The multiplicative consistency property of IFRPR is also investigated, and the consistency indexes for the three different levels of an IFRPR are defined. Additionally, similarity indexes of IFRPR are defined to measure the level of agreement among the group of experts. The consensus level is derived by combining both the consistency index and similarity index, and it is used to guide a feedback mechanism to support experts in changing their opinions to achieve a consensus solution with a high degree of consistency. Finally, a quantifier guided non-dominance possibility degree (QGNDPD) based prioritisation method to derive the final trust–consensus based solution is proposed.

1. Introduction

In the procedure of group decision-making (GDM), experts usually need to compare a finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ with respect to a single criterion, and construct preference relations. In general, there are two basic preference relations: multiplicative preference relation [3,35,39] and fuzzy preference relation [4,33]. In both cases, the preference relation elements represent the dominance of one alternative over another and take the form of exact numerical values. However, many decision making processes take place in an environment in which the information is not precisely known [1,11,12,20,34,36,46,50,53]. As a consequence, experts may feel more comfortable using an interval number rather than an exact crisp numerical value to represent their preference. Therefore, interval-valued fuzzy reciprocal preference relations (IFRPRs) [22,49] can be considered an appropriate representation format to capture experts' uncertain preference information. Indeed, the use of IFRPRs in GDM problems under uncertain environments has recently attracted the attention of many researchers [13,30,41,51].

In GDM problems, the individual preferences are aggregated to a collective one for deriving a solution. This is achieved by determining aggregation weights for each expert to compute the collective preference of the group from the individual preferences. As a consequence, one key issue that needs to be addressed in this type of decision making environment is how "weights of experts" should be derived. In most GDM models, the weights of expert are usually considered to be known beforehand or provided by a reliable source being therefore no part of the decision model design. However, in some cases, these assumptions may be unrealistic or improbable. Thus, it could be interesting to provide alternative ways to obtain such information.

Trust can reflect the actual reputation between experts [2] because it uses the history of an expert's actions or behaviour. Therefore, it should be taken into account as a reliable source to be used in deriving aggregation weights for individual experts. Social Network Analysis (SNA) methodology studies the relationships between social entities like members of a group, corporations or nations and it is a useful methodology to examine structural and locational properties such as: centrality, prestige and structural

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balance [18,37,40]. In this article, we focus on one type of social networks in which the users explicitly express their opinion on other users as trust statements. Furthermore, to represent the uncertainty or fuzziness of trust relationship between group experts, this article develops an interval-valued fuzzy SNA to define and measure the trust degree (TD) of individual experts.

Additionally to TD, consensus level (CL) has been previously considered another reliable source to derive the weights for individual experts in consensus models [5,7,26,42–45,47,49]. However, these consensus models are static in nature because they do not produce any type of rules to increase consensus when it is unacceptably low. Obviously, it is preferable that the group of experts achieve a high consensus level before aggregating individual preferences into a collective one. Recently, Chiclana et al. [8] and Herrera-Viedma et al. [25] investigated methodologies to develop feedback mechanisms to produce recommendations on how to increase consensus level. Inspired by these approaches, new consensus level (CL) and feedback mechanism for GDMs with IFRPRs are proposed.

Combining the two reliable sources representing the importance degree of experts, the trust degree (TD) and the consensus level (CL), a trust–consensus based approach to determine the weights of experts to use in aggregating individual IFRPRs into the collective one is proposed. Then, by applying the possibility degree (PD) of interval-valued fuzzy numbers (IFNs), a quantifier guided non-dominance possibility degree (QGNDPD) method is developed to derive the priority vector of the collective IFRPR.

The rest of paper is set out as follows: Section 2 introduces the multiplicative transitivity property and the corresponding definition of consistency for IFRPRs. In Section 3, the trust degree (TD) of experts is computed using SNA. A consensus model for GDM with IFRPRs is presented in Section 4, with special attention paid to the design of the consistency-consensus based feedback mechanism. Section 5 develops a process for deriving the collective IFRPR via the aggregation of the individual IFRPRs that is driven by a trust–consensus based methodology to determine the weights of experts. A quantifier guided non-dominance possibility degree (QGNDPD) method to exploit the collective IFRPR is also presented in this section. An analysis of the trust–consensus based model with respect to other GDM models is proposed in Section 6. Finally, conclusions are drawn in Section 7.

2. Consistency of interval-valued fuzzy reciprocal preference relations

Let X be a universe of discourse. A fuzzy set A on X is characterised by a membership function $\mu_A : X \to [0,1]$, and it is expressed as follows [53]:

$$A = \{(x, \mu_A(x));\ \mu_A(x) \in [0,1]\, \forall x \in X\} \tag{1}$$

Note that the membership grades of A are crisp numbers.

Given three alternatives $x_i,\ x_j,\ x_k$ such that x_i is preferred to x_j and x_j to x_k, the question whether the 'degree or strength of preference' of x_i over x_j exceeds, equals, or is less than the 'degree or strength of preference' of x_j over x_k cannot be answered by the classical preference modelling [9]. The introduction of the concept of fuzzy set as an extension of the classical concept of set when applied to a binary relation leads to the concept of a fuzzy relation. The adapted definition of a fuzzy reciprocal preference relation (FRPR) is the following one [4,33]:

Definition 1 (*Fuzzy Reciprocal Preference Relation (FRPR)*). A fuzzy reciprocal preference relation (FRPR) P on a finite set of alternatives $X = \{x_1, \ldots, x_n\}$ is characterised by a membership function $\mu_P : X \times X \longrightarrow [0,1]$, with $\mu_P(x_i, x_j) = p_{ij}$, verifying

$$\forall i,j \in \{1, \ldots, n\}:\ p_{ji} = 1 - p_{ij} \tag{2}$$

Membership functions are subject to uncertainty arising from various sources [15,17,32]. Klir and Folger [29, page 12] comment:

"...it may seem problematical, if not paradoxical, that a representation of fuzziness is made using membership grades that are themselves precise real numbers. Although this does not pose a serious problem for many applications, it is nevertheless possible to extend the concept of the fuzzy set to allow the distinction between grades of membership to become blurred."

Here Klir and Folger described blurring a fuzzy set to form an *interval-valued fuzzy set (IFS)* [14,16,28]:

Definition 2 (*Interval-Valued Fuzzy Set (IFS)*). Let $INT([0,1])$ be the set of all closed subintervals of $[0,1]$ and X be an universe of discourse. An interval-valued fuzzy set (IFS) $\widetilde{A}$ on X is characterised by a membership function $\mu_{\widetilde{A}} : X \to INT([0,1])$, and it is expressed as follows:

$$A = \{(x, \mu_{\widetilde{A}}(x));\ \mu_{\widetilde{A}}(x) \in INT([0,1])\, \forall x \in X\} \tag{3}$$

Given two interval numbers $\tilde{a}_1 = [a_1^-, a_1^+]$ and $\tilde{a}_2 = [a_2^-, a_2^+]$, the main interval arithmetic operations can be expressed in terms of the interval lower and upper bounds as follows [19]:

(1) $\tilde{a}_1 + \tilde{a}_2 = [a_1^-, a_1^+] + [a_2^-, a_2^+] = [a_1^- + a_2^-, a_1^+ + a_2^+]$.
(2) $\tilde{a}_1 - \tilde{a}_2 = [a_1^-, a_1^+] - [a_2^-, a_2^+] = [a_1^- - a_2^+, a_1^+ - a_2^-]$.
(3) $\tilde{a}_1 \cdot \tilde{a}_2 = [a_1^-, a_1^+] \cdot [a_2^-, a_2^+] = [(a_1a_2)^-, (a_1a_2)^+]$,

$$(a_1a_2)^- = \min\{a_1^-a_2^-, a_1^-a_2^+, a_1^+a_2^-, a_1^+a_2^+\}$$
$$(a_1a_2)^+ = \max\{a_1^-a_2^-, a_1^-a_2^+, a_1^+a_2^-, a_1^+a_2^+\}$$

(4) $\tilde{a}_1/\tilde{a}_2 = [a_1^-, a_1^+]/[a_2^-, a_2^+] = [(a_1/a_2)^-, (a_1/a_2)^+]$,

$$(a_1/a_2)^- = \min\{a_1^-/a_2^-, a_1^-/a_2^+, a_1^+/a_2^-, a_1^+/a_2^+\}$$
$$(a_1/a_2)^+ = \max\{a_1^-/a_2^-, a_1^-/a_2^+, a_1^+/a_2^-, a_1^+/a_2^+\}$$

provided that $0 \notin [a_2^-, a_2^+]$.

Note that real numbers $a \in \mathbb{R}$ can be represented in interval form as $[a,a]$. Two interval numbers $\tilde{a}_1 = [a_1^-, a_1^+]$ and $\tilde{a}_2 = [a_2^-, a_2^+]$ are equal if and only if $a_1^- = a_2^-$ and $a_1^+ = a_2^+$. An interval number $\tilde{a} = [a^-, a^+]$ is positive when $a^- \geqslant 0$. The product and division of positive interval numbers can be simplified as follows:

(3) $\tilde{a}_1 \cdot \tilde{a}_2 = [a_1^-, a_1^+] \cdot [a_2^-, a_2^+] = [a_1^-a_2^-, a_1^+a_2^+]$.
(4) $\tilde{a}_1/\tilde{a}_2 = [a_1^-, a_1^+]/[a_2^-, a_2^+] = [a_1^-/a_2^+, a_1^+/a_2^-]$, provided that $a_2^- > 0$.

The application of the concept of IFS to a FRPR leads to the concept of interval-valued fuzzy reciprocal preference relation (IFRPR) [22,49]:

Definition 3 (*Interval-Valued Fuzzy Reciprocal Preference Relation (IFRPR)*). An interval-valued fuzzy reciprocal preference relation (IFRPR) $\widetilde{P}$ on a finite set of alternatives $X = \{x_1, \ldots, x_n\}$ is characterised by a membership function $\mu_{\widetilde{P}} : X \times X \longrightarrow INT([0,1])$, with $\mu_{\widetilde{P}}(x_i, x_j) = \tilde{p}_{ij} = [p_{ij}^-, p_{ij}^+]$, verifying

$$\forall i,j \in \{1, \ldots, n\}:\ \tilde{p}_{ji} = 1 - \tilde{p}_{ij} \tag{4}$$

The above definition of IFRPR can be expressed in terms of the lower and upper bound of the interval-valued preference values as follows:

$$\forall i,j = 1,2,\ldots n:\ p_{ij}^- + p_{ji}^+ = p_{ij}^+ + p_{ji}^- = 1 \tag{5}$$