Knowledge-Based Systems 59 (2014) 132–141

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Interval-valued intuitionistic fuzzy continuous weighted entropy and its application to multi-criteria fuzzy group decision making

Feifei Jin, Lidan Pei, Huayou Chen *, Ligang Zhou

School of Mathematical Sciences, Anhui University, Hefei, Anhui 230601, China

ARTICLE INFO

Article history:
Received 18 March 2013
Received in revised form 15 January 2014
Accepted 16 January 2014
Available online 24 January 2014

Keywords:
Multi-criteria decision making
Group decision making
Interval-valued intuitionistic fuzzy set
Continuous weighted entropy
COWA operator

ABSTRACT

In this paper, we propose the interval-valued intuitionistic fuzzy continuous weighted entropy which generalizes intuitionistic fuzzy entropy measures defined by Szmidt and Kacprzyk on the basis of the continuous ordered weighted averaging (COWA) operator. It is shown that the continuous entropy of interval-valued intuitionistic fuzzy set is the average of the entropies of its interval-valued intuitionistic fuzzy values (IVIFVs). We also establish the programming model to determine optimal weight of criteria with the principle of minimum entropy. Furthermore, we investigate the multi-criteria group decision making (MCGDM) problems in which criteria values take the form of interval-valued intuitionistic fuzzy information. An approach to interval-valued intuitionistic fuzzy multi-criteria group decision making is given, which is based on the weighted relative closeness and the IVIFV attitudinal expected score function. Finally, emergency risk management (ERM) evaluation is provided to illustrate the application of the developed approach.

1. Introduction

The theory of fuzzy sets (FSs) put forward by Zadeh [1] has achieved a great success in various fields. Atanassov [2] introduced the concept of intuitionistic fuzzy sets (IFSs), which is the generalization of the FSs. The introduction of IFSs proved to be very meaningful and practical, and has been found to be highly useful to deal with vagueness. In IFSs, the data information is expressed by means of 2-tuples, and each 2-tuples is characterized by the degree of membership and non-membership. Furthermore, Atanassov and Gargov [3] introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs), whose components are intervals rather than exact numbers.

In order to calculate the aggregation values of the alternatives, a lot of works [4–7,25,28–30,32,33] have been done about the aggregation operators of the IVIFSs. Atanassov [4] proposed some operational laws for IVIFSs. Xu and Chen [5] developed some interval-valued intuitionistic fuzzy aggregation operators, such as the interval-valued intuitionistic fuzzy weighted averaging (IIFWA) operator, the interval-valued intuitionistic fuzzy ordered weighted averaging (IIFOWA) operator and the interval-valued intuitionistic fuzzy hybrid aggregation (IIFHA) operator, and then, they gave an application of the IIFHA operator to multiple attribute group decision making with interval-valued intuitionistic fuzzy information. Park et al. [6] presented a method for multi-person multi-attribute decision making based on the interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator and the interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator. Wei and Wang [7] investigated the interval-valued intuitionistic fuzzy ordered weighted geometric aggregation (IIFOWGA) operator and the interval-valued intuitionistic fuzzy hybrid geometric aggregation (IIFHGA) operator for multiple attribute group decision making under interval-valued intuitionistic fuzzy environment. Wang et al. [8] proposed an approach to multi-attribute decision making with incomplete attribute weighted information where individual assessments are provided as IVIFSs. Park et al. [9] presented an improved correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multi-attribute group decision-making problems with partially known attribute weight information.

As an important topic in the theory of fuzzy sets, entropy measures have been investigated widely from different points of view, such as decision making [10–12] and pattern recognition [13–15]. The fuzzy entropy was first introduced by Zadeh [16]. Moreover, Luca and Termini [17] presented the axioms with which the fuzzy entropy should comply, and defined the entropy of a fuzzy set based on Shannon's function. Szmidt and Kacprzyk [18] constructed the axiomatic requirements of intuitionistic fuzzy entropy measure and proposed a non-probabilistic-type entropy measure for IFSs based on the ratio of intuitionistic fuzzy cardinalities. Hung and Yang [19] gave their axiomatic definitions of entropy of IFS by exploiting the concept of probability.

* Corresponding author. Tel.: +86 136 156 90958.
E-mail addresses: shexian19880129@163.com (F. Jin), huayouc@126.com (H. Chen), shuiqiaozlg@126.com (L. Zhou).

http://dx.doi.org/10.1016/j.knosys.2014.01.014
0950-7051/

However, due to the increasing complexity of the social-economic environment and the lack of knowledge or data about the problem domain, the decision information may be provided with IVIFSs. For example, emergency risk management (ERM) evaluation aims evaluating and improving social preparedness and organizational ability of an emergency operating center (EOC) in identifying, analyzing and treating emergency risks to the community arising from emergency events [35]. Suppose that there are five pre-determined alternatives (emergency operating centers), the criteria associated with alternatives include energy, food, health and medical services, etc., the evaluation values may be given with IVIFSs because of fuzzy environment in the process of multi-criteria decision making. Therefore, it is necessary and important to extend the entropy measures to accommodate the situation with interval-valued intuitionistic fuzzy information.

Ye [20] proposed two entropy measures for IVIFSs and established an entropy weighted model to determine the entropy weights with respect to a decision matrix provided as IVIFSs. Wei et al. [21] developed an entropy measure for IVIFSs, which generalized three entropy measures for IFSs. Wu [22] introduced the concept of intuitionistic fuzzy weighted entropy, and gave a new method for intuitionistic fuzzy multi-criteria decision making problems.

Motivated by the concept of intuitionistic fuzzy weighted entropy proposed by Wu [22], we develop a new formula to calculate the entropy of an interval-valued intuitionistic fuzzy set based on the continuous ordered weighted averaging (COWA) operator [23], called interval-valued intuitionistic fuzzy continuous entropy, which generalizes intuitionistic fuzzy entropy measures defined by Szmidt and Kacprzyk [18]. Then, we introduce the concept of the interval-valued intuitionistic fuzzy continuous weighted entropy. According to the principle of minimum entropy, we establish the programming model to determine optimal weight of criteria. Moreover, we give an approach to multi-criteria group decision making based on the proposed continuous entropy measures under interval-valued intuitionistic fuzzy environment and the TOPSIS method, in which we utilize the weighted relative closeness for each alternative with respect to ideal alternative. The novelty of this weighted relative closeness is that it can take into account the decision makers' attitudinal character.

The rest of the paper is organized as follows. In Section 2, we briefly review some concepts of IFSs and IVIFSs. Section 3 presents the concepts of the interval-valued intuitionistic fuzzy continuous entropy and the interval-valued intuitionistic fuzzy continuous weighted entropy. In Section 4, an approach to interval-valued intuitionistic fuzzy multi-criteria group decision making is proposed, which is based on the interval-valued intuitionistic fuzzy continuous weighted entropy. Section 5 provides a numerical example of emergency operating center evaluation to illustrate the application of the developed method. In Section 6, we end the paper by summarizing the main conclusions.

2. Preliminaries

2.1. IFSs and the entropy of IFSs

In the following, we introduce some basic concepts related to IFSs and several methods for calculating entropy measures for IFSs.

Definition 2.1 [2]. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set. An intuitionistic fuzzy set (IFS) A over X is an object having the form:

$$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}, \quad (1)$$

where

$$\mu_A : X \to [0,1],\ \ \nu_A : X \to [0,1] \quad (2)$$

with the condition

$$0 \leqslant \mu_A(x_i) + \nu_A(x_i) \leqslant 1,\ \ \forall x_i \in X. \quad (3)$$

For each $x_i \in X$, the numbers $\mu_A(x_i)$ and $\nu_A(x_i)$ represent the membership and non-membership degrees of x_i to A, respectively.

For each IFS A in X, let $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i)$,which is called the hesitancy degree of x_i to A. It is obvious that $0 \leqslant \pi_A(x_i) \leqslant 1$, $x_i \in X$.

Let $\Gamma(X)$ be the set of all the IFSs on X.

The operations of IFSs [2] are defined as follows.

If $A \in \Gamma(X)$, $B \in \Gamma(X)$, then

$$A^c = \{\langle x_i, \nu_A(x_i), \mu_A(x_i)\rangle | x_i \in X\}, \quad (4)$$

$$A \cap B = \{\langle x_i, \min\{\mu_A(x_i), \mu_B(x_i)\}, \max\{\nu_A(x_i), \nu_B(x_i)\}\rangle | x_i \in X\}, \quad (5)$$

$$A \cup B = \{\langle x_i, \max\{\mu_A(x_i), \mu_B(x_i)\}, \min\{\nu_A(x_i), \nu_B(x_i)\}\rangle | x_i \in X\}. \quad (6)$$

For an IFS $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$, Szmidt and Kacprzyk [18] first axiomatized intuitionistic fuzzy entropy measure.

Definition 2.2. Suppose that I is a real-valued function $I: \Gamma(X) \to [0,1]$. I is an entropy measure of IFSs if it satisfies the following axiomatic requirements:

(1) $I(A) = 0$ if and only if A is a crisp set;
(2) $I(A) = 1$ if and only if $\mu_A(x_i) = \nu_A(x_i)$ for all $x_i \in X$;
(3) $I(A) = I(A^c)$;
(4) $I(A) \leqslant I(B)$ if A is less fuzzy than B, i.e., $\mu_A(x_i) \leqslant \mu_B(x_i)$ and $\nu_A(x_i) \geqslant \nu_B(x_i)$ for $\mu_B(x_i) \leqslant \nu_B(x_i)$ or $\mu_A(x_i) \geqslant \mu_B(x_i)$ and $\nu_A(x_i) \leqslant \nu_B(x_i)$ for $\mu_B(x_i) \geqslant \nu_B(x_i)$ for all $x_i \in X$.

Using the biggest cardinality (max-sigma-count) of IFSs, Szmidt and Kacprzyk also introduced an entropy measure for IFSs as follows:

Definition 2.3 [18]. The Szmidt and Kacprzyk entropy of an IFS $A \in \Gamma(X)$ is defined as

$$I_{SK}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\max count(A_i \cap A_i^c)}{\max count(A_i \cup A_i^c)}, \quad (7)$$

where

$A_i = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle\}$ and $\max count(A) = \sum_{i=1}^{n}(\mu_A(x_i) + \pi_A(x_i))$.

For an IFS A, Wang and Lei [25] gave a different entropy formula by

$$I_{WL}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\min\{\mu_A(x_i), \nu_A(x_i)\} + \pi_A(x_i)}{\max\{\mu_A(x_i), \nu_A(x_i)\} + \pi_A(x_i)}. \quad (8)$$

Huang and Liu [26] introduced the concept of vague fuzzy entropy. Based on the equivalence of two theories of vague sets and intuitionistic fuzzy sets [27], we can transform the vague fuzzy entropy into an intuitionistic fuzzy entropy formula [26] for an IFS A by the following equation:

$$I_{HL}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{1 - |\mu_A(x_i) - \nu_A(x_i)| + \pi_A(x_i)}{1 + |\mu_A(x_i) - \nu_A(x_i)| + \pi_A(x_i)}. \quad (9)$$

The entropy formulas (7)–(9) are introduced from different points of view. It is interesting to study their relations. Note that in [21], Wei et al. proved that $I_{SK}(A) = I_{WL}(A) = I_{HL}(A)$ for an IFS A.

2.2. IVIFSs and the operations between IVIFSs

In some cases, it is not appropriate to assume that the membership degrees for certain elements of IFSs are exactly defined.