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Optimal contracts for the agency problem with multiple uncertain information

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ABSTRACT

There is usually such a kind of agency problem where one principal authorizes one agent to perform more than one task at the same time. However, the potential output of each task cannot be exactly predicted in advance, so there exist simultaneously multiple types of uncertain information about the potential outputs of all the tasks. In this case, how to design the optimal contract and how to investigate the impacts of the diversity of uncertain information on such an optimal contract become important and challenging for decision makers. Motivated by this, to filter out the uncertainty in the possible incomes, we firstly focus on the optimal contract when both the two participators' potential incomes are measured by their respective expected incomes. Following that, as an important innovation, confidence level is introduced to quantify the degree of the agent's risk aversion, and the effects of the agent's attitude toward risk on the optimal contract and the principal's income are taken into account. Based on this view, two classes of uncertain agency models are developed, and then the sufficient and necessary conditions for the optimal contracts are presented with the detailed proofs and analyses. Compared with the traditional agency model, the innovations and advantages of the proposed work are briefly summarized, and the effectiveness of the work is further demonstrated by the computational results in a portfolio selection problem.

1. Introduction

Agency problems exist in all walks of social life, such as economics, finance, marketing and sociology. Once one party authorizes another one to perform some tasks, an agency problem appears between the two participators. The former is called principal, and the latter is agent. Generally speaking, there exists so complex uncertainty in agency problems that some information cannot be predicted by the two participators in advance, that is, the principal and the agent are both unknown about such information, referred to as incomplete information. For example, an investor authorizes a portfolio manager to invest his funds in more than one security, but the potential profit per share cannot be exactly predicted in advance, because there are several unpredictable and uncontrollable influencing factors in security market. Therefore, it is desirable for researchers to deal with agency problems in the case of incomplete information, especially in the case of multiple incomplete information. As we all know, the optimal actions of the two participators are usually regulated by the optimal contract, which is an equilibrium outcome between the principal and the agent after a successful negotiation. Intuitively, what the optimal contract includes can indicate the participators' optimal decisions in the agency relation. Furthermore, the optimal contract is usually associated with the participators' attitudes toward risk and their respective bargaining powers.

In the relevant literature, random variable has been often used to describe the incomplete information in agency problems based on probability theory. For example, the state-space formulation was firstly proposed by Wilson [26], Spence and Zechhauser [23], Ross [21] to describe the agency relation directly, but the solutions with some economic information cannot be obtained by this method. The parameterized distribution formulation was then presented by Mirrlees [16,17] and Holmstrom [5] to overcome this difficulty, and it has been a basic method to solve the agency problems. Following that, Groves [4] investigated an incentive issue of information that is necessary to induce team behavior by considering a team model of a general organization; Gilbert and Weng [3] studied a cost minimization problem in a service network by devising a strategy for allocating compensation and customers to the self-interested operators. The interested readers may also refer to the book [20] about the comprehensive development of agency theory in random environment.

With the development of fuzzy theory [2,13,19,28,29], fuzzy variable has been applied to measure the uncertainty in optimization problems, such as the fuzzy multi-period production planning problem [6], the multiple periodic factor prediction problem [15],

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the pricing decision problems in supply chain coordination [30,31] and the merchandize apportionment problem [32]. Just as stated in these results, fuzzy variable is applicable to characterize the subjective assessment about incomplete information in some particular situations. Following these results, Cui et al. [1] focused on the agency problem with fuzzy private information and established a fuzzy expected value model to solve it; Lan et al. [7] considered a nonlinear taxation problems with asymmetric information by presenting a bilevel fuzzy agency model.

As above, the existing literature depicted the incomplete information in agency problems by random variable or fuzzy variable. However, some information, such as duration time of one project, stock price and security income, cannot be exactly predicted by principals and agents in advance, thus resulting in the unknown frequencies. Therefore, probability theory is no longer applicable to characterize such kind of incomplete information, because the lack of the frequency leads to the unknowable probability distribution. In addition, some researchers apply fuzzy theory to handle the incomplete information via possibility measure or necessary measure, which describes the same issue from different perspectives. The possibility measure assesses the incomplete information in terms of affirmation, and the necessary measure does so in terms of disaffirmation. But, the use of them may lead to such a situation where the former overrates the possibility of the incomplete information to be correctly captured, while the latter underrates such a possibility. To overcome this difficulty by seeking for a more desirable method, a self-dual measure, called uncertain measure, has been introduced recently by Liu [8].

Uncertain measure is an alternative tool to characterize such incomplete information that has no any historical data and cannot be exactly predicted in advance. Liu [8] gave the basic concepts about uncertainty theory, such as uncertainty distribution, expected value, variance, and uncertain differential equations [22]. And then, Liu [11] proposed the measure inversion theorem, the linearity of expected value operator and operational law based on the concept of independence [9], Liu and Ha [14] deduced an important formula to calculate the expected values of monotone functions of uncertain variables. Following that, Liu [10] presented uncertain programming to model the optimization problems with uncertain parameters, and Yao [27] described the evolution of stock price in uncertain financial markets by mean-reverting stock model. The interested readers may refer to the book [12] for the comprehensive development of uncertainty theory. For simplicity, the incomplete information is called uncertain information when it is characterized by uncertain variable.

In the framework of uncertainty theory, Mu et al. [18] studied the implementation of the employment relationship problem between the enterprise and the rural migrant worker, and the enterprise's assessment on the rural migrant worker's own income at home was characterized as an uncertain variable because the income at home was privately possessed by the worker. The results showed that the rural migrant worker's optimal effort level decreased with his own income at home. Wang et al. [24] investigated an uncertain price discrimination problem in labor market, in which the employer did not know the employee's capability with certainty. Thus, an uncertain price discrimination model was developed by describing the employee's capability as an uncertain variable, and the optimal contract illustrated that both the productivity and the wage were strictly increasing with respect to the employee's capability.

Different from the work in the literature [18,24], this paper discusses such an agency problem with multiple types of uncertain information. Due to the uncertainty, the potential incomes of the two participators are so complex that they cannot be quantified directly. As a basic method, the expected income is employed to characterize the principal's future profit. Meanwhile, the agent's income is measured by two criterions, that is, the expected income and the income under his acceptable confidence level. The former focuses on the optimal contract when the two individuals' goals are quantified by the same criterion, and the latter pays more attention to the impact of the agent's attitude toward risk on the optimal contract, especially on the principal's optimal income. For this purpose, confidence level is introduced to quantify the degree of the agent's risk aversion, while the relevant literature just basically categorizes decision makers in three groups, that is, risk-averse, risk-neutral and risk-loving. As another advantage, the diversity of the uncertain information will not raise a multiple integral problem by expectation criterion in the framework of uncertainty theory, but such a computational difficulty is inevitable in the case of probability theory.

The rest of this paper is organized as follows. Section 2 recalls some fundamental concepts and formulas about uncertain variable. Section 3 discusses the agency problem with multiple uncertain information under two situations by proposing the uncertain agency models with expectation constraint and chance constraint, respectively. And then, the optimal contracts and the effects of the diversity of uncertain information on them are presented in detail. Section 4 summarizes the main innovations and advantages of the proposed work by comparing it with the traditional agency problem based on probability theory and the existing work in the relevant literature. Section 5 illustrates the validity of the proposed work by analyzing a portfolio selection problem between an investor and a portfolio manage. Section 6 concludes the paper.

2. Fundamental concepts

Let $(\Gamma, \mathcal{L}, \mathcal{M})$ be an uncertainty space, where Γ is a nonempty set, $\mathcal{L}$ a σ-algebra over Γ, and $\mathcal{M}$ an uncertain measure [8]. A measurable function $\xi : \Gamma \to \Re$ is called an uncertain variable if the set $\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$ is an event for any Borel set B. The uncertainty distribution Φ of an uncertain variable ξ is defined by $\Phi(x) = \mathcal{M}\{\xi \leqslant x\}$, $\forall x \in \Re$. Furthermore, if the uncertainty distribution $\Phi(x)$ is continuous, by the measure inversion theorem [11], then for any real number x,

$$\mathcal{M}\{\xi \geqslant x\} = 1 - \Phi(x). \tag{1}$$

The uncertainty distribution Φ of ξ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0,1)$. The inverse function Φ^{-1} is called the inverse uncertainty distribution of ξ. Based on inverse uncertainty distribution, the expected value of the uncertain variable ξ with an inverse uncertainty distribution $\Phi^{-1}(\alpha)$ is defined by [12]

$$\mathrm{E}[\xi] = \int_0^1 \Phi^{-1}(\alpha)\,\mathrm{d}\alpha.$$

Example 1. An uncertain variable ξ is called linear, if it has a linear uncertainty distribution [12]

$$\Phi(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{b-a}, & \text{if } a \leqslant x < b \\ 1, & \text{if } x \geqslant b, \end{cases}$$

denoted by $\mathcal{L}(a, b)$, where a and b are real numbers with $a < b$. The inverse uncertainty distribution of the linear uncertain variable $\mathcal{L}(a, b)$

$$\Phi^{-1}(\alpha) = (1-\alpha)a + \alpha b, \tag{2}$$

and the expected value