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# Type-1 OWA methodology to consensus reaching processes in multi-granular linguistic contexts

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ABSTRACT

A crucial step in group decision making (GDM) processes is the aggregation of individual opinions with the aim of achieving a "fair" representation of each individual within the group. In multi-granular linguistic contexts where linguistic term sets with common domain but different granularity and/or semantic are used, the methodology widely applied until now requires, prior to the aggregation step, the application of a unification process. The reason for this unification process is the lack of appropriate aggregation operators for directly aggregating uncertain information represented by means of fuzzy sets. With the recent development of the Type-1 Ordered Weighted Averaging (T1OWA) operator, which is able to aggregate fuzzy sets, alternative approaches to multi-granular linguistic GDM problems are possible. Unlike consensus models based on unification processes, this paper presents a new T1OWA based consensus methodology that can directly manage linguistic term sets with different cardinality and/or semantic without the need to perform any transformation to unify the information. Furthermore, the linguistic information could be assumed to be balanced or unbalanced in its mathematical representation, and therefore the new T1OWA approach to consensus is more general in its application than previous consensus reaching processes with muti-granular linguistic information. To test the goodness of the new consensus reaching approach, a comparative study between the T1OWA based consensus model and the unification based consensus model is carried out using six randomly generated GDM problems with balanced multi-granular information. When distance between fuzzy sets used in the T1OWA based approach is defined as the corresponding distance between their centroids, a higher final level of consensus is achieved in four out of the six cases although no significant differences were found between both consensus approaches.



## 1. Introduction

Decision making is an inherent activity of human beings. Everyday, human beings, consciously or unconsciously, make decisions about different aspects related to their life. Group decision making (GDM) has proven its usefulness as a decision methodology to address complex decisions in which the participation of experts from different areas may be interesting and even advisable. Moreover, in many of these decision making processes it is common to encounter problems where experts have to assess qualitative aspects that cannot easily be evaluated using precise quantitative assessments. In these cases the use of linguistic assessments seems be more appropriate to express experts' preferences. The fuzzy linguistic approach has proven its utility to deal with the imprecision and vagueness associated to qualitative information [1]. In this approach, qualitative aspects are represented by means of linguistic variables whose values are words rather than numbers. Concerning linguistic variables, semantic rules are defined in order to associate to each element of the universe of discourse its meaning. This interpretation of the meaning of a linguistic label is formally captured using the concept of fuzzy set, and therefore linguistic labels can formally be considered and represented as fuzzy subsets of their universe of discourse [1]. Another important aspect to be taken into account in the linguistic approach is the "granularity of uncertainty", i.e. the finest level of distinction among different quantifications of uncertainty as represented by the cardinality of the corresponding linguistic term set [2].

In GDM problems with experts belonging to different areas or with distinct levels of knowledge about the problem, it seems natural to expect that they will express opinions and/or preferences using different sets of linguistic terms and in general with different cardinality (granularity). Consequently, the development of

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adequate tools to manage and model multi-granular linguistic information becomes very important in the resolution of this type of GDM problem [3–9].

In a multi-granular linguistic context, the aggregation of elements from linguistic term sets of different cardinality and semantics is a challenging issue [10–12]. Different approaches have been proposed in the literature to address this and, among them, one of the most widely used requires a unification process methodology previous to the aggregation operation [6]. The unification process methodology is based on transformation functions with domain each one of the different linguistic term sets and same co-domain, known as the base linguistic term set (BLTS). Although transformation functions operate with the membership functions of the fuzzy sets used to represent the linguistic terms to be aggregated, such functions have been always subject of criticisms because they are not bijective and are subject to possible loss of information. Recently, Zhou et al. [13] proposed the Type-1 Ordered Weighted Average (T1OWA) operator that is able to directly aggregate type-1 fuzzy sets. The T1OWA operator, which is developed via the application of the extension principle to Yager's OWA operator [14], has been successfully proven to aggregate linguistic opinions in human decision making with linguistic weights [15–18]. This operator has the following main characteristics: (i) it allows the direct aggregation of different types of linguistic term sets – balanced or unbalanced sets, triangular or trapezoidal linguistic labels, etc.; (ii) the weighting vector consists of elements that can be crisp and precise numbers or fuzzy ones; (iii) it uses the whole membership function of the fuzzy sets to aggregate in the computation of the fuzzy aggregated value; and (iv) it allows the implementation of the concept of soft majority in the decision process if required. In summary, the use of the T1OWA operator in developing decision making models makes the current unification process superfluous and allows its direct application, i.e. there is no need to modify and/or adapt the model, to a wider range of decision making problems under uncertainty.

GDM problems generally involve situations of conflict among its experts, and therefore it is preferable that the set of experts reach consensus before applying a selection process to derive the decision solution [8,10,19,20]. Consensus is defined as the full and unanimous agreement of all the experts, a definition that is inconvenient in practice because it only allows differentiating between two states, namely, the existence and absence of consensus. The chances for reaching such a full agreement are rather low, while it is recognised that most real life situation unanimity is not necessary [21,22]. Thus, one key issue that needs to be addressed in a GDM problem is the evaluation of the level of consensus of the group of experts. Consensus is modelled mathematically via the use of similarity function measuring the concept of proximity of information [23]. In the linguistic model, the computation of similarity degrees between experts relies on the use of a distance function between the fuzzy sets representing their linguistic preferences. When the consensus level reaches a threshold value, agreed by the group of experts, the resolution process of the GDM is carried out; otherwise a feedback mechanism is activated, and personalised recommendations generated to support the individual experts, until the threshold level of consensus is achieved. The feedback recommendations will help the experts to identify the preference values that should be considered for changing. The idea of preserving as much as possible the initial experts' preferences [24] has also motived novel methodologies to reach consensus based on linear-programming based approaches that aim at minimising cost under the weighted averaging operator and OWA operators [25].

The aim of this paper is to present a new methodology to consensus reaching processes in multi-granular linguistic contexts based on the T1OWA operator. The new consensus reaching model allows the direct processing of the membership functions of the fuzzy sets modelling the linguistic information and therefore makes the unification process step currently used in developed models unnecessary. Furthermore, because the membership functions are nor required to fulfil extra conditions regarding their balanced or unbalanced distribution within the underlying domain of the variable used to measure preferences, nor they are required to be of the same shape type, the proposed methodology offers a greater degree of flexibility or generality in its application than existing models do. Having said this, a comparative study between the T1OWA based consensus model and the unification based consensus model is included using six randomly generated GDM problems with balanced multi-granular information. As it will be elaborated further later in the paper, when the distance between fuzzy sets in the T1OWA based approach is defined as the corresponding distance between their centroids, a higher final level of consensus is obtained in four out of the six cases studied, although no significant differences are found between both consensus approaches. Arguably, the T1OWA methodology can be used with guarantee in consensus reaching multi-granular linguistic decision making problems.

The rest of the paper is organised as follows. In Section 2 contains a short, but necessary for the set of the paper, review of concepts concerning multi-granular fuzzy linguistic GDM problems, the unification methodology of multi-granular linguistic information and the consensus reaching processes. Section 3 presents the T1OWA operator and its alpha-level fast implementation. Section 4 focuses on the presentation of the new T1OWA methodology to consensus reaching processes in multi-granular linguistic contexts. A comparative study between the new consensus methodology and the consensus methodology based on the unification process of preferences is presented in Section 5. Finally, some conclusions are pointed up in Section 6.

## 2. Preliminaries

To make the paper self-contained, the main concepts that will be used are introduced here.

### 2.1. Linguistic variable

A linguistic variable is formally represented by a 5-tuple $\langle L, T(L), U, S, M\rangle$ [1] where (i) $L$ is the name of the variable; (ii) $T(L)$ is a finite term set of (primary) labels or words (a collection of linguistic values); (iii) $U$ is a universe of discourse or base variable; (iv) $S$ is the syntactic rule which generates the terms in $T(L)$; and (v) $M$ is a semantic rule which associates with each linguistic value $X$ its meaning $M(X)$: $U \to [0,1]$. Usually, $T(L)$ is denoted as $L$ when there is no risk of confusion.

The semantic rule, also known as 'compatibility function' [1], associates with each element of the base variable its compatibility with each linguistic value. This interpretation of the meaning of a linguistic label coincides with that of a fuzzy set, and therefore linguistic labels can be considered and formally represented as fuzzy subsets of their base variable. Therefore, the nature of the base variable will dictate the general method to use when manipulating linguistic values. A non-numerical base variable makes the definition of the compatibility function 'difficult to be formalized in explicit terms' [1]. As a result, it turns out to be problematic when implemented at present in computer programmes. Thus, it is fair to say that most, if not all, important linguistic decision models in the literature assume that the base variable is a subset of the set of real numbers, and therefore numeric in nature. Indeed, these linguistic decision models usually start associating the linguistic