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Multiperson decision making with different preference representation structures: A direct consensus framework and its properties

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ABSTRACT

This study proposes a direct consensus framework for multiperson decision making (MPDM) with different preference representation structures (preference orderings, utility functions, multiplicative preference relations and fuzzy preference relations). In this framework, the individual selection methods, associated with different preference representation structures, are used to obtain individual preference vectors of alternatives. Then, the standardized individual preference vectors are aggregated into a collective preference vector. Finally, based on the collective preference vector, the feedback adjustment rules, associated with different preference representation structures, are presented to help the decision makers reach consensus. This study shows that the proposed framework satisfies two desirable properties: (i) the proposed framework can avoid internal inconsistency issue when using the transformation functions among different preference representation structures; (ii) it satisfies the Pareto principle of social choice theory. The results in this study are helpful to complete Chiclana et al.'s MPDM with different preference representation structures.

1. Introduction

In multiperson decision making (MPDM) problems, it is quite natural that different decision makers may have different experience, cultures and educational backgrounds. As a result, these decision makers may use different preference representation structures to express their individual preference information.

Chiclana et al. [6] initiated a notable MPDM model based on fuzzy preference relations, where the preference information can be represented by means of preference orderings, utility functions and fuzzy preference relations. Chiclana et al. [7] further incorporated multiplicative preference relations in the MPDM model. Herrera et al. [22] proposed a multiplicative MPDM model involving three kinds of preference representation structures (preference orderings, utility functions, multiplicative preference relations), assuming the multiplicative preference relations as the uniform element of the preference representation structures. Dong et al. [16] presented a linguistic MPDM model based on linguistic preference relations, integrating fuzzy preference relations, different types of multiplicative preference relations and multigranular linguistic preference relations. Moreover, Herrera et al. [23], Herrera and Martínez [26], Herrera-Viedma et al. [31], Mata et al. [35], Chen and Ben-Arieh [5] and Jiang et al. [32] introduced several methods to solve the MPDM problems with multi-granularity linguistic evaluation information. In [30], Herrera-Viedma et al. proposed a consensus model for the MPDM problem with different preference representation structures. Palomares et al. [39] proposed an attitude-driven web consensus support system for heterogeneous group decision making.

Several desirable properties have been proposed in the MPDM model with different preference representation structures. Chiclana et al. [9] and Dong et al. [13] discussed the conditions under which the reciprocity property is maintained in the aggregation of fuzzy preference relations using the ordered weighted average (OWA) operator [46] guided by a relative linguistic quantifier [47]. Meanwhile, in the above MPDM models, the *internal consistency* is a key issue. The internal consistency refers to the ranking among alternatives derived from the transformed preference representation structure is the same one from the original preference representation structure. Chiclana et al. [7,8] and Dong et al. [16] studied the conditions under which the internal consistency is maintained.

Inspired by the MPDM model initiated by Chiclana et al. [6] and the corresponding consensus model presented in Herrera-Viedma et al. [30], and also inspired by the direct approach presented in Herrera et al. [25], this study proposes a direct consensus framework for MPDM problems with different preference representation structures (preference orderings, utility functions, multiplicative preference relations and fuzzy preference relations). In the direct consensus framework, the individual selection methods, associated with different preference representation structures, are used to

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obtain individual preference vectors of alternatives. Then, the standardized individual preference vectors are aggregated into a collective preference vector. Finally, based on the collective preference vector, the feedback adjustment rules, associated with different preference representation structures, are presented to help the decision makers reach consensus. The results in this study are helpful to complete Chiclana et al.'s MPDM with different preference representation structures, based on the following reasons:

(i) The proposed framework can avoid internal inconsistency issue when using the transformation functions among different preference representation structures.
(ii) It satisfies the Pareto principle of social choice theory.

The rest of this paper is organized as follows. Section 2 introduces the preliminary knowledge regarding four kinds of preference representation structures (preference orderings, utility functions, multiplicative preference relations and fuzzy preference relations) and the OWA operator. A direct consensus framework for MPDM problems with different preference representation structures is proposed, and the differences between our proposal and models presented in [6,7,30] are analyzed in Section 3. Following this, the selection process is designed in Section 4. Subsequently, Section 5 proposes the consensus processes to help the decision makers reach consensus. Two desirable properties are presented in Section 6, and an illustrative example is provided in Section 7. Finally, concluding remarks are included in Section 8.

2. Preliminaries: Four kinds of preference representation structures and OWA operator

This section introduces four kinds of preference representation structures and the OWA operator.

2.1. Preference representation structures

Let $X = \{x_1, x_2, \ldots, x_n\} (n \geqslant 2)$ be a finite set of alternatives. These alternatives have to be classified from best to the worst, according to the preference information provided by a set of decision makers, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geqslant 2)$.

This study assumes that the decision makers' preference information over the set of alternatives X may be represented in one of the following four formats (see Definitions 1–4).

Definition 1 (*Preference orderings* [43]). A vector $O = (o_1, o_2, \ldots, o_n)^T$ is called a preference ordering, where $o_i (o_i = 1, 2, \ldots, n)$ denotes the positional order of alternative x_i in $X = \{x_1, x_2, \ldots, x_n\}$.

Definition 2 (*Utility functions* [44]). A vector $U = (u_1, u_2, \ldots, u_n)^T$ is called a utility function, where $u_i \in [0, 1]$ represents the utility evaluation value given by a decision maker to the alternative x_i.

Definition 3 (*Multiplicative preference relations* [41]). A matrix $A = (a_{ij})_{n \times n}$ is called a multiplicative preference relation if $a_{ij} \times a_{ji} = 1$ and $a_{ij} > 0$ for $\forall i, j$, where a_{ij} indicates a ratio of the preference intensity of alternative x_i to that of x_j.

Definition 4 (*Fuzzy preference relations* [37,44]). A fuzzy preference relation on a set of alternatives X is represented by a matrix $P = (p_{ij})_{n \times n}$, where $p_{ij} \in [0, 1]$ denotes the preference degree of the alternative x_i over x_j. A fuzzy preference relation usually assumed to be additive reciprocal, i.e., $p_{ij} + p_{ji} = 1$, $\forall i, j$.

2.2. OWA operator

Let $\{a_1, a_2, \ldots, a_l\}$ be a set of values to aggregate. The OWA operator is defined as [46]

$$OWA(a_1, a_2, \ldots, a_l) = \sum_{k=1}^{l} \lambda_k b_k. \tag{1}$$

where b_k is the k th largest value in $\{a_1, a_2, \ldots, a_l\}$, and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)^T$ is an associated weight vector, such that $\lambda_i \in [0, 1]$ and $\sum_{i=1}^{l} \lambda_i = 1$.

In [47], Yager suggested an effective method to compute $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)^T$ using linguistic quantifiers, which, in the case of a non-decreasing proportional quantifier Q, is given by this expression:

$$\lambda_i = Q(i/l) - Q((i-1)/l), \quad i = 1, 2, \ldots, l. \tag{2}$$

$Q(r)$ can be represented as [48]:

$$Q(r) = \begin{cases} 0, & r < a, \\ \frac{r-a}{b-a}, & a \leqslant r \leqslant b, \\ 1, & r > b, \end{cases} \tag{3}$$

with $a, b, r \in [0, 1]$.

There are several common relative linguistic quantifiers, such as *all*, *most*, *at least half* and *as many as possible*, where the parameters (a, b) are (0, 1), (0.3, 0.8), (0, 0.5) and (0.5, 1), respectively. When a fuzzy linguistic quantifier Q is used to compute the weights of the OWA operator, it is symbolized by OWA_Q.

3. Proposed framework

This section proposes a direct consensus framework for the MPDM with different preference representation structures. And the differences between our proposal and the models presented in Chiclana et al. [6,7] and Herrera-Viedma et al. [30] are also analyzed.

3.1. Direct consensus framework

Let X and E be as earlier. Let E^U, E^O, E^A and E^P be four subsets of E, representing decision makers whose preference information on X are expressed by utility functions, preference orderings, multiplicative preference relations, and fuzzy preference relations, respectively. Without loss of generality, this study assumes that $E^U = \{e_1, e_2, \ldots, e_{m_1}\}$, $E^O = \{e_{m_1+1}, e_{m_1+2}, \ldots, e_{m_2}\}$, $E^A = \{e_{m_2+1}, e_{m_2+2}, \ldots, e_{m_3}\}$ and $E^P = \{e_{m_3+1}, e_{m_3+2}, \ldots, e_m\}$.

In MPDM models (e.g., [24,27,30]), there are two processes to implement before obtain a final solution, namely: (1) the selection process; and (2) the consensus process. Inspired by these two processes, we proposed a direct framework for MPDM problems with four kinds of preference representation structures. This direct framework is presented in Fig. 1. In this framework, the selection process and the consensus process are used.

(1) Selection process.
In this process, we obtain the standardized individual preference vectors and the standardized collective preference vector. The implementation of this selection process deals with a two-step procedure.
(i) Obtaining individual preference vectors.
In this step, the individual selection methods, associated with different preference representation structures, are used to obtain individual preference vectors.
(ii) Obtaining a collective preference vector.
In this step, the individual preference vectors are transformed into standardized individual preference vectors.