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Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information

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ABSTRACT

Hesitant fuzzy set (HFS), which allows the membership degree of an element to a set represented by several possible values, is considered as a powerful tool to express uncertain information in the process of multi-attribute decision making (MADM) problems. In this paper, we develop a novel approach based on TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) and the maximizing deviation method for solving MADM problems, in which the evaluation information provided by the decision maker is expressed in hesitant fuzzy elements and the information about attribute weights is incomplete. There are two key issues being addressed in this approach. The first one is to establish an optimization model based on the maximizing deviation method, which can be used to determine the attribute weights. According to the idea of the TOPSIS of Hwang and Yoon [1], the second one is to calculate the relative closeness coefficient of each alternative to the hesitant positive-ideal solution, based on which the considered alternatives are ranked and then the most desirable one is selected. An energy policy selection problem is used to illustrate the detailed implementation process of the proposed approach, and demonstrate its validity and applicability. Finally, the extended results in interval-valued hesitant fuzzy situations are also pointed out.

1. Introduction

Multi-attribute decision making (MADM), which addresses the problem of making an optimal choice that has the highest degree of satisfaction from a set of alternatives that are characterized in terms of their attributes, is a usual task in human activities. In classical MADM, the assessments of alternatives are precisely known [2,3]. However, because of the inherent vagueness of human preferences as well as the objects being fuzzy and uncertain, the attributes involved in decision making problems are not always expressed in real numbers, and some are better suited to be denoted by fuzzy values, such as interval values [4–6], linguistic variables [7,8], intuitionistic fuzzy values [9–12], and hesitant fuzzy elements (HFEs) [13,14], just to mention a few. Since Zadeh [15] first proposed the basic model of fuzzy decision making based on the theory of fuzzy mathematics in 1965, fuzzy MADM has been receiving more and more attention. Many methods for MADM, such as the TOPSIS method [16–18], the maximizing deviation method [19–21], the gray relational analysis method [22], the VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) method [23–25], the PROMETHEE (Preference Ranking Organisation METHod for Enrichment Evaluations) method [26], and the ELECTRE (ELimination Et Choix Traduisant la REalité) method [27,28], have been extended to take different types of attribute values into account, such as interval values, linguistic variables, and intuitionistic fuzzy values. All of the above methods, however, have not yet been accommodated to fit the hesitant fuzzy assessments provided by the decision makers (DMs).

Hesitant fuzzy set (HFS) [13,14], which has been introduced by Torra and Narukawa as an extension of fuzzy set [15], describes the situations that permit the membership of an element to a given set having a few different values, which is a useful means to describe and deal with uncertain information in the process of MADM. For example, to get a reasonable decision result, a decision organization, which contains a lot of DMs, is authorized to estimate the degree that an alternative should satisfy a criterion. Suppose that there are three cases, some DMs provide 0.3, some provide 0.5, and the others provide 0.6, and these three parts cannot persuade each other, thus the degree that the alternative should satisfy the criterion can be represented by a HFE $\{0.3, 0.5, 0.6\}$. It is noted that the HFE $\{0.3, 0.5, 0.6\}$ can describe the above situation more objectively than the interval-value fuzzy set $[0.3, 0.6]$, because the degrees that the alternative should satisfy the criterion are not the convex combination of 0.3 and 0.6, or the interval between 0.3 and 0.6, but just three possible values [31]. So, the use of hesitant

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fuzzy assessments makes the DMs' judgments more reliable and informative in decision making. Xia and Xu [29] developed some aggregation operators for hesitant fuzzy information, and gave their application for solving MADM problems under hesitant fuzzy environment. Xu and Xia [30] gave a detailed study on distance and similarity measures for HFSs and proposed an approach based on distance measures for MADM problems. Xia et al. [31] also proposed some other hesitant fuzzy aggregation techniques and applied them in group decision making. Yu et al. [32] proposed a hesitant fuzzy Choquet integral operator and applied it in MADM under hesitant fuzzy environment in which the weight vector of attributes is exactly known. Wei [33] also developed some prioritized aggregation operators for hesitant fuzzy information, and developed some models for hesitant fuzzy MADM problems in which the attributes are in different priority levels. Yu et al. [34] proposed the generalized hesitant fuzzy Bonferroni mean to solve MAGDM problems where the attributes are correlative under hesitant fuzzy environment. More recently, Qian et al. [35] generalized the HFSs using intuitionistic fuzzy sets in group decision making framework. The generalized HFS is fit for the situations when the DMs have a hesitation among several possible memberships under uncertainty. Chen et al. [36] also generalized the concept of HFS to that of interval-valued hesitant fuzzy set (IVHFS) in which the membership degrees of an element to a given set are not exactly defined, but denoted by several possible interval values, and meanwhile developed an approach to group decision making based on interval-valued hesitant preference relations in order to consider the differences of opinions between individual DMs. Obviously, most of these papers put their emphasis on the extensions of the aggregation techniques in MADM under hesitant fuzzy scenarios. However, when using these techniques, the associated weighting vector is more or less determined subjectively and the decision information itself is not taken into consideration sufficiently; and more importantly, a significant pitfall of the aforementioned methods is the need for the information about attribute weights being exactly known.

In fact, in the process of MADM with hesitant fuzzy information, we often encounter the situations where the attribute values take the form of HFEs, and the information about attribute weights is incompletely known or completely unknown because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain [37]. Considering that the existing methods cannot be suitable for dealing with such situations, in this paper, we propose a novel approach to objectively determine the attribute weights and sort the alternatives under the conditions that the attribute weights are completely unknown or partly known, and the attribute values take the form of HFEs. To do so, we organize the paper as follows: In Section 2, we review some concepts related to HFSs and IVHFSs. Section 3 develops a novel approach based on TOPSIS and the maximizing deviation method for solving the MADM problem with hesitant fuzzy information. Section 4 extends our results to interval-valued hesitant fuzzy environment. Section 5 gives the application of the developed approach to MADM involving energy policy selection and makes some comparison analysis. The paper finishes with some concluding remarks in Section 6.

2. Some basic concepts

Hesitant fuzzy set [13,14], as a generalization of fuzzy set, permits the membership degree of an element to a set presented as several possible values between 0 and 1, which can better describe the situations where people have hesitancy in providing their preferences over objects in the process of decision making.

Definition 1 (*13,14*). Let X be a reference set, a hesitant fuzzy set (HFS) A on X is defined in terms of a function $h_A(x)$ when applied to X returns a subset of $[0,1]$, i.e.,

$$A = \{< x, h_A(x) > | x \in X\} \tag{1}$$

where $h_A(x)$ is a set of some different values in $[0,1]$, representing the possible membership degrees of the element $x \in X$ to A. For the sake of simplicity, Xia and Xu [29] called $h_A(x)$ a hesitant fuzzy element (HFE).

It is noted that the number of values for different HFEs may be different, and the values are usually out of order, then we can arrange them in any order for convenience. Suppose that we arrange the values of a HFE h in an increasing order, and let $h_{\sigma(i)}$ ($i = 1, 2, \ldots, l_h$) be the ith smallest value in h. For two HFEs α and β, let $l = \max\{l_\alpha, l_\beta\}$, where l_α and l_β are respectively the numbers of values in the HFEs α and β. In order to more accurately calculate the distance between two HFSs, Xu and Xia [38] suggested that we should extend the shorter one until both of them have the same length when we compare them with $l_\alpha \neq l_\beta$, and they gave the following regulations:

If $l_\alpha < l_\beta$, then α should be extended by adding the minimal value in it until it has the same length with β; If $l_\alpha > l_\beta$, then β should be extended by adding the minimal value in it until it has the same length with α. At the same time, we can extend the shorter one by adding any value in it which mainly depends on the DMs' risk preferences. Optimists anticipate desirable outcomes and may add the maximum value, while pessimists expect unfavorable outcomes and may add the minimal value.

Although Xu and Xia's extension rule is very reasonable, it does not consider the situation when the DM is assumed to be risk-neutral. We now develop a new method, which can reveal the DM's risk preference (including risk-averse, risk-neutral, and risk-seeking) with a parameter η, to extend the shorter HFE until both of them have the same length when we compare them with $l_\alpha \neq l_\beta$.

Definition 2. Assume a HFE $h = \{h_{\sigma(i)} | i = 1, 2, \ldots, l_h\}$, and stipulate that h^+ and h^- are the maximum and minimum values in the HFE h, respectively; then we call $\bar{h} = \eta h^+ + (1-\eta)h^-$ an extension value, where $\eta (0 \leqslant \eta \leqslant 1)$ is the parameter determined by the DM according his/her risk preference.

Therefore, we can add different values to the HFE using η according the DM's risk preference. If $\eta = 1$, then the extension value $\bar{h} = h^+$, which indicates that the DM's risk preference is risk-seeking; while if $\eta = 0$, then $\bar{h} = h^-$, which means that the DM's risk preference is risk-averse. It is clear that Xu and Xia's extension rule is consistent with our extension rule when $\eta = 1$ and $\eta = 0$. Moreover, when the DM's risk preference is risk-neutral, we can add the extension value $\bar{h} = \frac{1}{2}(h^+ + h^-)$, i.e., $\eta = \frac{1}{2}$. Apparently, the parameter η provided by the DM reflects his/her risk preference which can affect the final decision results.

Meanwhile, Torra [14] indicated that the envelope of a HFE is an intuitionistic fuzzy value (IFV), which is shown as follows:

Definition 3 [14]. Given a HFE h, we define the IFV $A_{env}(h)$ as the envelope of h, where $A_{env}(h)$ can be represented as $(h^-, 1-h^+)$, with $h^- = \min\{\gamma | \gamma \in h\}$ and $h^+ = \max\{\gamma | \gamma \in h\}$.

Based on the above operational laws and the principle of extension, Xu and Xia [38] defined the hesitant Euclidean distance for HFEs:

$$d_1(\alpha, \beta) = \sqrt{\frac{1}{l}\sum_{i=1}^{l} |\alpha_{\sigma(i)} - \beta_{\sigma(i)}|^2} \tag{2}$$