Knowledge-Based Systems 24 (2011) 146–153

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

An extended TOPSIS for determining weights of decision makers with interval numbers

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ARTICLE INFO

Article history:
Received 2 February 2010
Received in revised form 26 July 2010
Accepted 26 July 2010
Available online 3 August 2010

Keywords:
Multiple attribute group decision making (MAGDM)
Weight of decision maker
TOPSIS
Interval number
Air quality

ABSTRACT

In this paper, we develop a method for determining weights of decision makers under group decision environment, in which the each individual decision information is expressed by a matrix in interval numbers. We define the positive and negative ideal solutions of group decision, which are expressed by a matrix, respectively. The positive ideal solution is expressed by the average matrix of group decision and the negative ideal solution is maximum separation from positive ideal solution. The separation measures of each individual decision from the ideal solution and the relative closeness to the ideal solution are defined based on Euclidean distance. According to the relative closeness, we determine the weights of decision makers in accordance with the values of the relative closeness. Finally, we give an example for integrated assessment of air quality in Guangzhou during 16th Asian Olympic Games to illustrate in detail the calculation process of the developed approach.

1. Introduction

Multiple attribute decision making (MADM) occurs in a variety of actual situations, such as economic analysis, strategic planning, forecasting, medical diagnosis, venture capital and supply chain management. The increasing complexity of the socioeconomic environment makes it less and less possible for a single decision maker (DM) to consider all relevant aspects of a problem [1]. As a result, many decision making processes, in the real world, take place in group settings. Moving from single DM's setting to group members' setting would lead to a great deal of complexity of the analysis. For example, consider that these DMs usually come from different specialty fields, and thus each DM has his/her unique characteristics with regard to knowledge, skills, experience and personality, which implies that each DM usually has different influence in overall decision result, i.e., the weights of DMs are different. Therefore, how to determine the weights of DMs will be an interesting and important research topic.

At present, many methods have been proposed to determine the weights of DMs. French [2] proposed a method to determine the relative importance of the group's members by using the influence relations, which may exist between the members. Theil [3] proposed a method based on the correlation concepts when the member's inefficacy is measurable. Keeney and Kirkwood [4] and Keeney [5] suggested the use of the interpersonal comparison to determine the scales constant values in an additive and weighted social choice function. Bodily [6] and Mirkin [7] proposed two approaches which use the eigenvectors method to determine the relative importance of the group's members. Brock [8] used a Nash bargaining based approach to estimate the weights of group members intrinsically. Ramanathan and Ganesh [9] proposed a simple and intuitively appealing eigenvector based method to intrinsically determine the weights of group members using their own subjective opinions. Martel and Ben Khélifa [10] proposed a method to determine the relative importance of group's members by using individual outranking indexes. Van den Honert [11] used the REMBRANDT system (multiplicative AHP and associated SMART model) to quantify the decisional power vested in each member of a group, based on subjective assessments by the other group members. Jabeur and Martel [12] proposed a procedure which exploits the idea of Zeleny [13] to determine the relative importance coefficient of each member. By using the deviation measures between additive linguistic preference relations, Xu [14] gave some straightforward formulas to determine the weights of DMs.

Many of literatures mentioned above described the individual decision information by a multiplicative preference matrix. Until now there has been little investigation of the weights of DMs based on individual decision information, in which the attribute values are given as observations in nonnegative real numbers, and the DMs have their subjective preferences on alternatives.

By considering the fact that, in some cases, determining precisely the exact values of the attributes is difficult and that, as a result of this, their values are considered as intervals. Therefore, in this article, we shall discuss the weights of DMs based on

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doi:10.1016/j.knosys.2010.07.014

technique for order performance by similarity to ideal solution (TOPSIS) [15] with interval numbers.

The rest of the paper is organized as follows: Section 2 reviews multiple attribute group decision making (MAGDM) and the TOPSIS technique, the basic idea and main contributions of the developed method in this paper are presented. The preliminaries, including comparing and ranking interval numbers, are given in Section 3. The developed approach and its algorithm to determine the weights of DMs are presented in Section 4. Section 5 makes two comparisons between the proposed method in this paper and the literature of other methods. In Section 6, we illustrate our proposed algorithmic method with an example. Conclusions appear in Section 7.

2. Literature survey

In this part we review the MAGDM, which has become an important part of modern decision science [16–19]. The decision information provided by the DMs may take the various representation formats in group decision making problems, such as exact numerical values [20–22], interval numbers [23–26], fuzzy numbers [27–30], fuzzy linguistic [31,32], rough set theory [33] and evidence theory [34]. In this paper, we will focus on the proposed group TOPSIS model in order to determine the weights of DMs.

TOPSIS, one of known classical MADM method, was first developed by Hwang and Yoon [15] for solving a MADM problem. TOPSIS technique is a hot research topic, which has received a great deal of attention from researchers [35–39]. The basic idea of TOPSIS is rather straightforward. It originates from the concept of a displaced ideal point from which the compromise solution has the shortest distance. Hwang and Yoon [15] further propose that the ranking of alternatives will be based on the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS) or nadir. TOPSIS simultaneously considers the distances to both the PIS and the NIS, and a preference order is ranked according to their relative closeness, and a combination of these two distance measures. The PIS/NIS, as a benchmark of TOPSIS method, is expressed by a vector. The traditional TOPSIS is limited to compare vectors of alternatives (with respect to attributes) with the vector of PIS/NIS. However, this comparison can not reflect DM's overall decisional level, which is expressed by a decision matrix (see Eq. (5)). Suppose that $X_1, X_2, \ldots, X_t$ are the decision matrixes of k ($k = 1, 2, \ldots, t$) DMs. This article intend to extend the PIS/NIS to a matrix X, which is a benchmark of $X_1, X_2, \ldots, X_t$. The decisional level of kth DM is measured by the Euclid distance between X_k and X, and the weight of kth DM is determined by his/her decisional level.

Jahanshahloo et al. [40] have extended the concept of TOPSIS to develop a methodology for solving MADM problems with interval data. Ye and Li [41] extended the TOPSIS technique for solving MAGDM problems with interval data, in which the DMs' weights are same. Sayadi et al. [42] developed an extension of VIKOR method [43,44] for MAGDM problem with interval numbers, in which the DMs' weights are also same. To overcome this limitation of same weights of DMs, we report a further extension of TOPSIS method in MAGDM environment with interval numbers, in which the DMs' weights are different. This paper focuses on determining the weights of DMs in MAGDM environment with interval data. The paper has the following main contributions:

1. The extended TOPSIS technique is also called group TOPSIS with interval data in this article. For the given individual decision matrixes, the PIS of group opinion is depicted by a matrix, in which every element is expressed in average of each individual decision interval; similarly, the NIS of group opinion is also depicted by a matrix, in which the decision information is expressed in maximum separation from the corresponding interval of positive case. The ranking of DMs (based on their decision matrixes) will be based on the shorter distance from the PIS and the farther from the NIS. That is, a DM's decision matrix is closer to the PIS and farther from the NIS, and then the DM is more weight.
2. In this paper, each DM has a decision matrix. The weight of DM is determined by both distances, which one is between the DM's decision matrix and the PIS, another is between the DM's decision matrix and the NIS. The second contribution of this paper is that this paper extends alternatives'/vectors' ranking based on ideal solutions to DMs'/matrixes' ranking based on ideal solutions. Here, the former ideal solutions are vectors while the latter ideal solutions are matrixes. TOPSIS technique focuses on the set of alternatives including PIS and NIS, while the extended TOPSIS technique focuses on the set of decision matrixes including PIS and NIS. Furthermore, the extended TOPSIS is clear in algorithm and without loss of information in aggregation, which no investigation has been devoted to.
3. The proposed method is suitable for determining the weights of attributes of group decision making when the exchange takes place between corresponding positions of DMs and attributes in each individual decision matrix. In this sense, the third contribution of this paper is that the proposed method not only can determine the weights of DMs, but also can determine the weights of attributes.

The TOPSIS method introduces two "reference" points: PIS and NIS in order to ranking of alternatives. The extended TOPSIS method in this paper, a key issue is determination of two "reference" points (or a benchmark) of all individual decision matrixes for comparison of the decisional levels among DMs. The reasons why the PIS is defined as the average matrix of group decision are that: (1) the PIS is the maximum compromise (in mean sense) among all individual decisions of group; (2) the "average" is adopted as the final decision (outcome) of group in most of the situations where a group decision must be taken. For example, for a teaching competition participated by young teachers in a university, if there is t DMs, the final score of each competitor is the average of t scores given by the DMs; and (3) the NIS is the maximum individual regret (the farthest distance from PIS).

TOPSIS method is suitable for cautious (risk avoider) DM(s), because the DM(s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible [42]. The developed approach in this paper assigns high weights to those DMs if the DMs want to have maximum group utility (majority/group), and minimum individual risk (minority/individual) in mean sense.

In order to realize the idea above, in the following, we will establish an extended TOPSIS model with interval data in a group decision environment.

3. Preliminaries

In the following, we first review the notion of the nonnegative interval number and some operational laws.

Definition 1 [45]. Let $a = [a^l, a^u] = \{x | 0 < a^l \leqslant x \leqslant a^u\}$, then a is called a nonnegative interval number. Especially, a is a nonnegative real number, if $a^l = a^u$.

Note: For convenience of computation, throughout this paper, all the interval arguments are nonnegative interval numbers.

Definition 2 [45,46]. Let $a = [a^l, a^u]$, $b = [b^l, b^u]$ are interval numbers and $\lambda \geqslant 0$, then