Available online at www.sciencedirect.com
ScienceDirect
Knowledge-Based Systems 19 (2006) 430–437
Knowledge-Based SYSTEMS
www.elsevier.com/locate/knosys

Risk and confidence analysis for fuzzy multicriteria decision making

Norman Fenton *, Wei Wang

Risk Analysis and Decision Research Group, Department of Computer Science, Queen Mary University of London, Mile End Road, London E1 4NS, UK

Received 8 February 2005; accepted 24 March 2006
Available online 16 June 2006

Abstract

Recent research has recognised that multicriteria decision making (MCDM) should take account of uncertainty, risk and confidence. This paper takes this research forward by using linguistic variables and triangular fuzzy numbers to model the decision maker's (DM) risk and confidence attitudes in order to define a more complete MCDM solution. To illustrate the computation process and demonstrate the feasibility of the results we use a travel problem that has been used previously to assess MCDM techniques. The results show that the method is useful for tackling imprecision and subjectivity in complex, ill-defined and human-oriented decision problems.

Keywords: Multicriteria decision making; Triangular fuzzy numbers; Linguistic variables

1. Introduction

Multicriteria decision making (MCDM) refers to screening, prioritising, ranking, or selecting a set of *alternatives* (also referred to as "candidates" or "actions") under usually independent, incommensurate or conflicting *criteria* [2,16,28]. We will use the following example (also used in [15,31]) to illustrate the concepts and methods throughout:

Example. We have to reach the airport from our home to catch an airplane. The MCDM problem here is to select an appropriate travel type from three alternatives: *Car*, *Taxi* and *Train*. Our criteria are *price*, *journey time*, and *comfort*.

An MCDM problem is characterized by (a) the *ratings* of each alternative with respect to each criteria and (b) the *weights* given to each criteria. Classical MCDM methods assume that the ratings of alternatives and the weights of criteria are *crisp* numbers. Increasingly, this is recognized as unrealistic. In the above example, the decision maker (DM) will be unable to assign a crisp number for the journey time of a car since this value is influenced by many factors. Generally, uncertainties arise from: unquantifiable information, incomplete information, unobtainable information, and partial ignorance [8].

Since classical MCDM methods cannot handle problems with such imprecise information, the representation and interpretation of "uncertainty" and human-related subjective preference is needed [40]. The use of probabilistic methods for this purpose in MCDM has been explored in [15,31], but fuzzy set theory [38] seems to have been the most commonly used method. The general use of fuzzy set theory in MCDM is explored in [3,24,25,37], while specific fuzzy MCDM methods can be found in [4,6,8–12,14,27,32–34]. Fuzzy decision making with partial preference information has been explored in [5,18,22,25,30]. In [35–37], Yager included fuzzy methods, probabilistic information as well as the DM's attitudes and preferences for decision-making under uncertainty.

In this paper, we first introduce the general fuzzy MCDM approach (Section 2). Then we focus on the two dimensions where we believe the DM's attitude is most subjective: *risk* (Section 3) and *confidence* (Section 4). We handle risk by extending the so-called linguistic approach [1,13,17,21,39] that has previously been explored with fuzzy MCDM in [7,10–12,20,32–34]. The linguistic approach is

* Corresponding author. Tel.: +44 20 7882 7860; fax: +44 20 8980 6533.
E-mail addresses: norman@dcs.qmul.ac.uk (N. Fenton), weiwang@dcs.qmul.ac.uk (W. Wang).

0950-7051/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.knosys.2006.03.002

an approximate way to represent natural words or sentences used in human judgment and perception. Linguistic decision analysis [4,17,18,26] transforms the linguistic description of the DM into a mathematical model to provide a flexible framework for solving decision problems. To handle confidence we use the fuzzy α cut concept [19] in addition to a linguistic approach. Our method for ranking the performance of alternatives is based on the kind of two-phase approach adopted in [8,25,40]. The first phase is to aggregate performance of the ratings of the alternatives under the criteria. The second phase is to rank alternatives with respect to aggregated performances.

2. General fuzzy MCDM approach

First we describe the general approach to fuzzy MCDM without considering risk attitudes and confidence.

2.1. Problem formulation and definitions

A general multicriteria decision problem with m alternatives A_i ($i = 1, \ldots, m$) and n criteria C_j ($j = 1, \ldots, n$) can be concisely expressed as:

$$D = [x_{ij}] \text{ and } W = (w_j), \text{ where } i = 1, \ldots, m \text{ and } j = 1, \ldots, n. \tag{1}$$

Here D is referred to as the *decision matrix* (where the entry x_{ij} represents the rating of alternative A_i with respect to criterion C_j), and W as the *weight vector* (where w_j represents the weight of criterion C_j). In general we classify criteria as either:

- *benefit criteria* (where the higher the value of x_{ij} the better it is for the DM) or
- *cost criteria* (where the lower the value of x_{ij} the better it is for the DM).

Because we wish to consider *fuzzy*, as opposed to *crisp*, values in D and W we shall use the notation:

$$\tilde{D} = [\tilde{x}_{ij}] \text{ and } \tilde{W} = (\tilde{w}_j), \tag{2}$$

whereby $\tilde{x}_{ij}$ represents the fuzzy rating of alternative A_i with respect to criterion C_j, and $\tilde{w}_j$ represents the fuzzy weight of criterion C_j. In particular, an intuitively easy and effective approach to capturing the expert's uncertainty about the value of an unknown number is a triangular fuzzy number:

Definition. A triangular fuzzy number $\tilde{a}$ is defined by a triplet (a_1, a_2, a_3). The membership function is defined as [19]:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a_1)/(a_2 - a_1), & a_1 \leqslant x \leqslant a_2, \\ (a_3 - x)/(a_3 - a_2), & a_2 \leqslant x \leqslant a_3, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The triangular fuzzy number is based on a three-value judgment: the minimum possible value a_1, the most possible value a_2 and the maximum possible value a_3.

Table 1
Decision matrix and weight vector

	Price(Pounds; 0.3)	Journey time(min; 0.5)	Comfort([1,10]; 0.2)
Car	(9, 10, 12)	(70, 100, 120)	(4, 5, 6)
Taxi	(20, 24, 25)	(60, 70, 100)	(7, 8, 10)
Train	(15, 15, 15)	(70, 80, 90)	(1, 4, 7)

Example. Table 1 shows the decision matrix and weight vector for the travel problem introduced in Section 1. In this example the criteria *price* and *journey time* are cost criteria measured in pounds and minutes respectively. The criterion *comfort* is a value criterion measured on a scale from 1 to 10. The ratings in the decision matrix are expressed as triangular fuzzy numbers (so, for example, the car journey to the airport most typically costs 10 pounds but it can be as low as 9 and as high as 12). For simplicity the weights are crisp numbers summing to 1 (usually the DM is able to express the weights in this way).

2.2. Normalization

To deal with criteria on different scales, we apply a normalization process. Specifically, we normalize the fuzzy numbers in the decision matrix as the performance matrix:

$$\tilde{P} = [\tilde{p}_{ij}], \tag{4}$$

where

$$\tilde{p}_{ij} = \begin{cases} \left(\frac{x_{ij1}}{M}, \frac{x_{ij2}}{M}, \frac{x_{ij3}}{M}\right), & M = \max_i x_{ij3}, \quad Cj \text{ is benefit criterion} \\ \left(\frac{N - x_{ij3}}{N}, \frac{N - x_{ij2}}{N}, \frac{N - x_{ij1}}{N}\right), & N = \max_i x_{ij3}, \quad Cj \text{ is cost criterion} \end{cases}$$

This method preserves the ranges of normalized triangular fuzzy numbers to $[0, 1]$.

Example. The performance matrix for the decision matrix of Table 1 is calculated by (4) and shown in Table 2.

2.3. Weighting the criteria

We construct the weighted performance matrix by multiplying the weight vector by the decision matrix as:

$$\tilde{P}^w = [\tilde{p}^w_{ij}], \tag{5}$$

where $p^w_{ij1} = w_{j1} p_{ij1}$, $p^w_{ij2} = w_{j2} p_{ij2}$, $p^w_{ij3} = w_{j3} p_{ij3}$, $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.

Example. The running example is shown in Table 3.

Table 2
Performance matrix

	Price (Pounds; 0.3)	Journey time (min; 0.5)	Comfort ([1, 10]; 0.2)
Car	(0.520, 0.600, 0.640)	(0.000, 0.167, 0.417)	(0.400, 0.500, 0.600)
Taxi	(0.000, 0.040, 0.200)	(0.167, 0.417, 0.500)	(0.700, 0.800, 1.000)
Train	(0.400, 0.400, 0.400)	(0.250, 0.333, 0.417)	(0.100, 0.400, 0.700)