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Pinning cluster synchronization in an array of coupled neural networks under event-based mechanism☆

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ABSTRACT

Cluster synchronization is a typical collective behavior in coupled dynamical systems, where the synchronization occurs within one group, while there is no synchronization among different groups. In this paper, under event-based mechanism, pinning cluster synchronization in an array of coupled neural networks is studied. A new event-triggered sampled-data transmission strategy, where only local and event-triggering states are utilized to update the broadcasting state of each agent, is proposed to realize cluster synchronization of the coupled neural networks. Furthermore, a self-triggered pinning cluster synchronization algorithm is proposed, and a set of iterative procedures is given to compute the event-triggered time instants. Hence, this will reduce the computational load significantly. Finally, an example is given to demonstrate the effectiveness of the theoretical results.

1. Introduction

Recently, synchronization of complex dynamical networks has been widely explored by researchers in various fields of physics, mathematics, engineering, biology and sociology (Cao, Sivasamy, & Rakkaiyappan, 2015; Cao & Wan, 2014; Chen, Liu, & Lu, 2007; Huang, Ho, & Lu, 2015; Li, Ho, & Lu, 2013; Lu & Ho, 2010; Lu, Ho, & Wu, 2009; Lu, Zhong, Li, Ho, & Cao, 2015; Lu et al., 2014; Watts & Strogatz, 1998; Wu, 2007; Xia & Cao, 2011). As a typical complex network, neural networks have also been intensively studied in the past years due to the broad applications (Cao, Alofi, Al-Mazrooei, & Elaiw, 2013; Liu, Ho, Yu, & Cao, 2014; Liu, Wang, & Liu, 2009; Song & Zhao, 2014; Tang, Wang, Gao, Qiao, & Kurths, 2014; Wang, Liu, Liu, & Shi, 2010; Wu, Shi, Su, & Chu, 2014). It can be observed that a synchronous behavior plays an important role in a proper functioning (e.g., information transmission and pattern recognition) of many neural networks (Hoppensteadt & Izhikevich, 2000). Complete synchronization of coupled neural networks has been studied extensively in past few years (Cao, Chen, & Li, 2008; Cao & Wan, 2014; Liu et al., 2009; Tang, Gao, Lu, & Kurths, 2014; Wen, Zeng, & Huang, 2012; Yang, Cao, & Lu, 2013). In Cao et al. (2008), based on the Lyapunov functional method and Kronecker product properties, several sufficient conditions are established to ensure the global exponential synchronization of coupled delayed neural networks. In Cao and Wan (2014), a single inertial BAM neural network with time-varying delays and external inputs is studied and several sufficient conditions are proposed for the global exponential stability of the equilibrium by using matrix measure. In Yang et al. (2013), delay-dependent sufficient synchronization criteria are derived for an array of coupled neural networks with random coupling strengths and mixed delays.

Another interesting problem on complex network synchronization is driving a network of interacting nodes onto a well defined reference trajectory. These kinds of problems are usually called pinning synchronization of complex networks, in which a feedback control law is applied to a small subset of the nodes (Chen et al., 2007; Huang, Ho, Lu, & Kurths, 2015; Lu, Ding, Lou, & Cao, 2015; Lu, Zhong, Huang, & Cao, in press; Tang, Gao et al., 2014). In Chen et al. (2007), the authors showed that a network with a rooted spanning

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tree can be synchronized by a single controller. In Tang, Gao et al. (2014), pinning synchronization of nonlinear dynamical networks with multiple stochastic disturbances is investigated. In Huang, Ho, Lu, and Kurths (2015), a new fuzzy complex networks with partial and discrete-time couplings model is proposed and pinning synchronization conditions of the proposed model are presented.

As a special kind of synchronization phenomenon, cluster synchronization means that the synchronization occurs within one group, but there is no synchronization among different groups. Recently, cluster synchronization of complex networks has received increasing attention because of its application in biological science and communication engineering (Cao & Li, 2009; Ji, Peron, Menck, Rodrigues, & Kurths, 2013; Liu & Chen, 2011; Su et al., 2013; Wu, Zhou, & Chen, 2009; Xia & Cao, 2011). In Xia and Cao (2011), different mechanisms (identical and non-identical self-dynamics) leading to clustering synchronization in diffusively coupled networks are discussed in detail. In Wu et al. (2009) and Liu and Chen (2011), pinning control strategies are proposed to steer a dynamical network to an expected cluster synchronization pattern. In Yoshioka (2005), it is demonstrated that cluster synchronization can be realized in an ensemble of neurons interacting through chemical synapses. The study of cluster synchronization phenomena is significant to the theoretical research on brain science and related practical applications (Cao & Li, 2009; Krüger, 1991; Yoshioka, 2005). Hence, in this paper, cluster synchronization problem of coupled neural networks will be studied. In the past few years, some works have been devoted to study cluster synchronization of coupled neural networks (Cao & Li, 2009; Li & Cao, 2011; Song & Zhao, 2014; Yoshioka, 2005). In Cao and Li (2009), by constructing a special coupling matrix, several sufficient criteria for cluster synchronization in an array of coupled neural networks are derived. The main results of Cao and Li (2009) are further extended to the stochastic delayed neural networks in Li and Cao (2011). It should be noted all previous mentioned works are based on continuous-time state information transmission.

In networked environment, successful and efficient communication among nodes is the key factor for dynamical systems to achieve desired collective behaviors. Due to the limited bandwidth of the communication channel among the nodes, it is necessary to save energy as much as possible. Recently, a novel communication protocol, namely, event-triggered control (see e.g. Li, Liao, Huang, and Zhu (2015), Mazo and Tabuada (2011), Tabuada (2007)), was developed to provide an effective methodology that satisfies the energy constraints of the system. Collective behaviors (e.g. synchronization and consensus) of complex network under event-triggered communication mechanism have become a hot research subject (Gao, Liao, & Li, 2014; Hu, Cao, Hu, & Guo, 2015; Li, Liao, Chen, Hill, & Huang, 2015; Lu, Han, & Chen, 2015; Zou, Wang, Gao, & Liu, 2015). Some interesting work on synchronization of complex networks under event-triggered communication strategy was shown in Li, Liao, Chen et al. (2015) and Lu et al. (2015). Effective event-triggered conditions are designed to achieve the synchronization of the considered network models. In Zou et al. (2015), state estimation of a class of complex networks is investigated under event-based information transmission. In Gao et al. (2014), a new distributed event-triggered mechanism for pinning control synchronization of complex networks is presented. In Hu et al. (2015), cluster synchronization of complex networks via event-triggered strategy under stochastic sampling is investigated and sufficient conditions for cluster synchronization are presented according to the Lyapunov stability theory. Unfortunately, the exclusion of Zeno behavior is not strictly proved in Gao et al. (2014) and Hu et al. (2015).

Motivated by above statements, we aim to study the pinning cluster synchronization of coupled neural networks by a novel event-triggered mechanism. Under event-triggered mechanism, some controllers will be pinned to certain selected nodes in coupled neural networks to realize expected cluster synchronization. The main difficulty of this paper is how to propose distributed event-triggered schemes to realize expected cluster synchronization and meanwhile exclude the Zeno behavior. Specifically, the main contributions of this paper are listed as follows:

- Based on the work of Cao and Li (2009) and Gao et al. (2014), the distributed event-triggered schemes are proposed to realize the expected cluster synchronization of coupled neural networks. Distributed event-triggered scheme means the event-triggered condition is verified by each agent only based on each own and received neighboring agents' information, i.e., only local information is used. Compared with Gao et al. (2014), our event-triggered schemes can exclude the Zeno behavior, i.e., only a finite number of events are triggered in any finite time period.
- To further reduce the computational load of the nodes, a distributed self-triggered algorithm is proposed for the considered model. The algorithm only updates at some specially appointed time instants. It is proved that the expected cluster synchronization can be achieved and Zeno behavior can be excluded simultaneously under the proposed self-triggered algorithm.

The organization of the remaining part is given as follows. In Section 2, some preliminaries are summarized, and coupled neural networks model under event-triggered mechanism is formulated. In Section 3, cluster synchronization analysis under the proposed event-triggered mechanism is presented in detail. In Section 4, a distributed self-triggered algorithm is proposed to realize cluster synchronization of the coupled neural networks. In Section 5, a numerical example is given to show the effectiveness of our theoretical results. In Section 6, concluding remarks are drawn.

Notation: The standard notation will be used in this paper. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ represent n-dimensional Euclidean space and the set of $m \times n$ real matrices, respectively. $\mathbb{N}$ represents the set of nonnegative integers. $\|\cdot\|$ represents the Euclidean norm for a vector or a matrix. I_n represents the n-dimensional identity matrix. $X > 0 (X < 0)$ denotes the symmetric matrix X is positive (negative) definite. The superscript "T" represents the transpose. The notation $[a, b]$ means the closed interval with endpoints a and b. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ are respectively the maximum and minimum eigenvalues of matrix A. Kronecker product is denoted by $\otimes$.

2. Preliminaries and problem formulation

In this section, we will first give some basic definitions and lemmas and then present the coupled neural networks model under event-triggered mechanism.

2.1. Preliminaries

Definition 1. A network with N nodes is said to realize cluster synchronization, if the N nodes are split into several clusters $G_1, G_2, \ldots, G_k$, such as, $\{G_1 = (1, 2, \ldots, m_1),\ G_2 = (m_1+1,\ m_1+2, \ldots, m_2), \ldots, G_k = (m_{k-1}+1,\ m_{k-1}+2, \ldots, m_k),\ m_0 = 0,\ m_k = N,\ m_{j-1} < m_j,\ j = 1, 2, \ldots, k\}$ such that

- the nodes in the same cluster synchronize with each other, i.e., for the states $x_i(t)$ and $x_j(t)$ of arbitrary nodes i and j in the same cluster, $\lim_{t\to+\infty} \|x_i(t) - x_j(t)\| = 0$ holds.
- the nodes in the different clusters do not synchronize, i.e., for the states $x_i(t)$ and $x_j(t)$ of arbitrary nodes i and j in different clusters, $\lim_{t\to+\infty} \|x_i(t) - x_j(t)\| \neq 0$ holds.